

# Sovereign Debt Tolerance with Potentially Permanent Costs of Default\*

Marcos Chamon<sup>†</sup>

IMF

Francisco Roldán<sup>‡</sup>

IMF

February 2024

## Abstract

We investigate the effect of uncertainty about the nature of output costs of sovereign default on debt tolerance. While the theoretical literature assumes output losses lasting until market access is restored, the empirical evidence points to persistent effects, and output may not return to its pre-default trend. We include such uncertainty in a model of sovereign default and find that it can significantly boost equilibrium debt levels. We also consider a government which is averse to this type of uncertainty and seeks robust decision rules. We calibrate the model to match evidence on the output trajectory around debt restructuring episodes and infer output costs of about the size found in the empirical literature, alongside significant uncertainty about their permanence and a strong desire for robustness.

**JEL Classification** E43, E44, F34, H63

**Keywords** Sovereign debt, default, debt tolerance, permanent costs, robustness

---

\*For comments and suggestions we thank our discussant Francesco Giavazzi, Tamon Asonuma, Stelios Fourakis, Javier García-Cicco, Leonardo Martinez, Ignacio Presno, Francisco Roch, as well as participants at the IMF's 24th Jacques Polak Annual Research Conference in honor of Ken Rogoff and UdeSA. Zhuo Chen provided excellent research assistance. The views expressed in this paper are those of the authors and should not be attributed to the IMF, its Executive Board, or its Management.

<sup>†</sup>e-mail: [mchamon@imf.org](mailto:mchamon@imf.org)

<sup>‡</sup>e-mail: [froidan@imf.org](mailto:froldan@imf.org)

## INTRODUCTION

A fundamental question in the sovereign debt literature, and one to which Rogoff made substantial contributions, is what supports debt repayments. In the absence of international enforcement of sovereign debts, costs of default, actual and perceived, physical and reputational, determine how much debt can be sustained as well as the terms at which countries borrow.

This paper focuses on how the possibility of permanent scarring effects of a default heightens the ex-ante costs of restructuring. Motivated by the lack of conclusive empirical evidence one way or the other, we consider uncertainty about how permanent such costs are. We draw implications for the amount of sustainable debt in equilibrium and find that the possibility of permanent costs has a large impact on “debt tolerance” (Reinhart, Rogoff, and Savastano, 2003), especially when combined with realistic risk attitudes for the debtor country.

In two influential papers, Bulow and Rogoff (1989a) show the limits of reputation and exclusion alone to sustain otherwise unenforceable payments on sovereign debt. Bulow and Rogoff (1989b) propose the inclusion of other costs affecting the economy, such as direct sanctions interfering with trade. The idea that countries sustain debt because defaults entail costs for the real economy still underpins most of the literature on sovereign debt, even if their source has shifted away from sanctions.<sup>1</sup> Costs can also involve broader general reputational spillovers affecting the rest of the economy, as in Cole and Kehoe (1998). Most models in the theoretical literature assume an output cost of default which lasts until credit market access is restored.

It is well documented that economic growth declines around the time of restructurings or defaults (Borensztein and Panizza, 2009) but the exact nature of such costs is not so clear. Subsequent studies with different methodologies (De Paoli et al., 2009; Asonuma et al., 2023; Farah-Yacoub et al., 2022; Cerra and Saxena, 2008) find growth slowdowns following a debt restructuring, some of which are quite persistent.

In practice, countries go to great lengths to remain current on their debts. If anything, they tend to wait until restructurings come “too late” (IMF, 2013), which suggests large perceived costs of restructurings, even though we do not observe much in the way of direct punishments, certainly not in the magnitude of the defaulted debt.

We evaluate the effect of potentially permanent costs within a sovereign default framework

---

<sup>1</sup>For surveys of the literature on sustainable public debt and sovereign default, see the handbook chapters by Aguiar and Amador (2014), Aguiar, Chatterjee, Cole, and Stangebye (2016), D’Erasmus, Mendoza, and Zhang (2016), and Martinez, Roch, Roldán, and Zettelmeyer (2023).

à la [Eaton and Gersovitz \(1981\)](#) with trend growth combined with uncertainty about the type of costs associated with default. In the model, a government chooses whether to repay its debt without knowing if defaulting would trigger a transitory or permanent decline in output. Moreover, we consider a government who does not fully trust its prior about the probability of a transitory cost and seeks robust decision rules as in [Hansen and Sargent \(2001\)](#).<sup>2</sup>

To obtain decision rules which are robust to different, unidentified misspecifications of its prior on the output cost persistence, the government entertains plausible distortions of it and evaluates each action's payoff under the worst one, the so-called worst-case model. While robustness is in general technically equivalent to risk-sensitive Epstein-Zin preferences ([Anderson, Hansen, and Sargent, 2003](#)), we leverage the flexibility of robustness to emphasize aversion towards uncertainty about the permanence of default costs in particular.<sup>3</sup>

These departures from the canonical model allow us to match reduced-form evidence on the trajectory of output following a debt restructuring. Our calibration identifies a degree of robustness and a probability of permanent costs which contribute to an increase of about a quarter in debt tolerance. Making the costs permanent for sure leads to an increase in debt tolerance of almost a half relative to fully transitory costs. Conceptually, a higher probability of permanent costs and a higher degree of robustness both contribute to debt tolerance, but with different implications for the observed output trajectory following a typical default. Finally, we also find that the robust government sustains higher levels of debt (as if permanent costs were more likely) but does not default less often.

The remainder of the paper is organized as follows. Section 2 discusses some of the possible costs associated with a restructuring and reviews the findings from the empirical literature on the costs of default. Section 3 presents the model and defines the equilibrium, while Section 4 describes its calibration. Section 5 discusses the quantitative results of the model. Section 6 concludes.

## 2. OUTPUT LOSSES AND OTHER COSTS AROUND RESTRUCTURINGS

The empirical literature estimates sizable impacts of defaults on economic activity, but deciding whether such effects are permanent is a statistically daunting task.<sup>4</sup> [Borensztein and Panizza](#)

---

<sup>2</sup>Robustness is also featured in [Pouzo and Presno \(2016\)](#) and [Roch and Roldán \(2023\)](#) in the context of sovereign debt, but on the lender side.

<sup>3</sup>For a discussion of robustness to “structured uncertainty,” see [Hansen and Sargent \(2015\)](#).

<sup>4</sup>Following most of the literature we use the term default loosely to refer to any distressed debt restructuring.

(2009) estimate a 2.6 percentage point decline in growth in the first year of the episode, but that effect (on growth) is short lived with no statistically significant effect on lagged variables. [Levy Yeyati and Panizza \(2011\)](#) also find in quarterly data that much of the contraction actually takes place before the default event in question. [Asonuma and Trebesch \(2016\)](#) show that restructurings that take place preemptively before payments are missed tend to be quicker and have better growth outcomes than those that take place after a default. [Asonuma et al. \(2023\)](#) estimate local projections on the level of log GDP relative to pre-crisis trend to investigate the persistence of a level effect. The adverse effects tend to be mild and temporary for preemptive restructurings, but large and persistent for restructurings involving a default (which historically has been the case in most restructurings). They estimate GDP to remain 4 percent below its pre-crisis trend five years after the default, which suggests these losses are not reversed.<sup>5</sup> [Farah-Yacoub et al. \(2022\)](#) use synthetic controls to estimate the economic and social costs of default on a long historical sample. They estimate a cumulative 8.4 percentage point gap within the first three years, with substantial scarring effects even after a decade.

Most of the literature has focused on external debt restructurings. One notable exception is [Reinhart and Rogoff \(2011\)](#), which show that the output decline after domestic restructurings is much larger than for external ones. Domestic debt restructurings were relatively rare during the 1980s and early 1990s, but have become more common as countries have started to rely more on domestic financing ([IMF, 2021](#)). Domestic restructurings can more directly lead to major financial disruption which can by itself entail significant output costs ([Pérez, 2018](#)). But following most of the literature, our paper focuses on external debt restructurings.

There is a wide range of outcomes following a debt restructuring. Some countries do experience mild and temporary declines, while for others the impact is deeper and more protracted. A few even experience a growth acceleration. [Figure 1](#) plots the evolution of the log of GDP relative to the pre-restructuring trend for different countries experiencing an external debt restructuring.<sup>6</sup> Time is measured relative to the restructuring event which takes place at  $t = 0$ . [Figure A.1](#) in the Appendix presents the same information in calendar time.

The left panel covers Emerging Market economies (EMs) and the right one, Low-Income Countries (LICs).<sup>7</sup> Each line traces the evolution of a country from 5 years prior to its restruc-

---

<sup>5</sup>Most sources of catch-up growth such as pent-up demand should have already materialized in that horizon.

<sup>6</sup>The events are based on the [Asonuma and Trebesch](#) dataset which includes both restructurings that take place after payments are missed (default) as well as distressed exchanges that take place preemptively prior to payments being missed. The latter is less frequent, but has become relatively more common over time.

<sup>7</sup>We define these two groups based on whether they are covered by the IMF-World Bank LIC Debt Sustainability Framework or the one for market access countries. The sample is listed in [Appendix A](#).

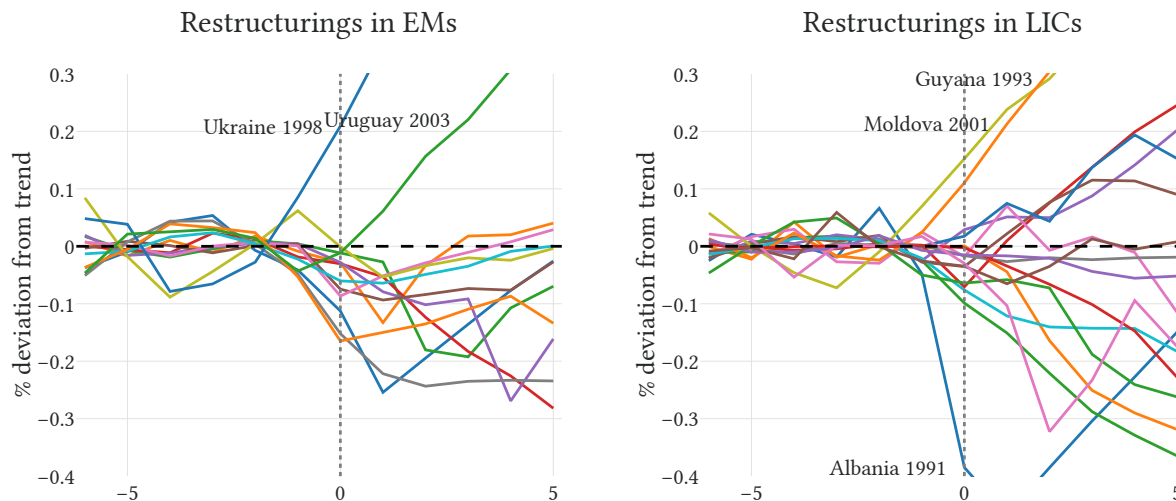


FIGURE 1: GROWTH OUTCOMES AROUND DEBT RESTRUCTURINGS

*Note:* Outlier trajectories censored and marked

turing to 10 years afterwards. When there are multiple restructurings within that time frame, we count each event separately. The log GDP series is detrended using a linear time trend based on the 5 years prior to the restructuring.<sup>8</sup> Hence by construction, the series hovers around zero prior to the restructuring, but can deviate significantly after that event.

There are fundamental differences between EM and LIC restructurings, especially in the historical sample. The official sector plays a much larger role in LICs, and many of those countries remained deeply insolvent until their debt crises was eventually resolved as part of the Heavily Indebted Poor Countries (HIPC) initiative. HIPC, combined with the Multilateral Debt Relief Initiative (MDRI) provided deep debt writedowns (enough to bring debt down to 150 percent of exports, sometimes even less). This helps explain the weak performance of LICs prior to the late 1990s/early 2000s, and their relatively strong performance afterwards. Given the particular features of LIC restructurings, we focus our attention on the sample of EMs, as shown in Figure 1A.

The growth performance of EMs tends to be weaker following restructurings, which were much more common in the 1980s (see Figure A.1 in the Appendix for the calendar time dimension of these events). There are several cases where GDP remains well below pre-crisis trends, reflecting the severity of that systemic event and the long time it took for its eventual resolution.

<sup>8</sup>A linear trend is preferable to using more sophisticated filters, such as HP, which would use observations after the crisis to inform its trend. One alternative could be to use one-sided filters using only backward-looking data, but for the purposes of this illustration a simple linear trend suffices.

It is also worth noting that external financial conditions were particularly unfavorable at that time (due to very high U.S. interest rates), and that many of the affected countries had deep structural problems (for example, very closed economies particularly in Latin America). From the 1990s onwards, we see relatively fewer restructuring episodes. We still see quite a few cases where GDP remains below trend, but there are also a number of cases where the lines remain closer to zero (which again, indicates the pre-crisis trend) or where the initial decline starts to revert sooner. There were a number of important innovations in the debt restructuring technology over time, including the adoption of Collective Action Clauses (CACs) on bonds and eventually enhanced CACs which allowed aggregation of votes across series (making it more difficult for a creditor to hold out when there was widespread support from other creditors to a restructuring). [IMF \(2020\)](#) takes stock of developments in sovereign debt restructurings involving the private-sector creditors and notes that compared with previous periods, recent restructurings have generally proceeded smoothly and quicker, and were more likely to be preemptive.

On balance, the output costs are substantial in the historical sample. There are a number of steps a debtor can take to minimize those costs, such as collaboratively engaging with creditors towards restructuring before payments are missed. But there are many things beyond the debtor's control, and restructurings are inherently complicated processes. Thus, even these costs have come down in expectation, there is still a tangible risk that things could go wrong and the costs turn out to be large. That cost can be further amplified by the risk and ambiguity aversion of the debtor, and help sustain more debt in equilibrium.

## 2.1 *Other costs*

Our focus has been on credit market exclusion combined with output costs, since these are easier to directly map into a model. As [Bulow and Rogoff \(1989a\)](#), [Bulow and Rogoff \(1989b\)](#), and [Rogoff \(2022\)](#) point out, credit exclusion in itself cannot yield a large enough cost to sustain observed levels of debt, which is why researchers tend to combine it with direct costs on productivity or income. But there are other potential sources of costs.

Direct sanctions have been proposed as another channel that could help sustain debt. Such punishments could translate into a decline in output, financial exclusion, and depending on the nature of the sanctions have a direct effect on welfare over and beyond the decline in output. In practice, we do not observe much in the way of such punishments on debtors. Even high profile cases of litigation involving asset seizures involve sums that are nowhere near the order of magnitude of the debt claims. The main example of a successful and disruptive litigation is Argentina

vs NML Capital, where a 2014 ruling prevented the debtor from repaying the restructured bonds (from its 2001 default) without repaying the litigants in full. Such litigation can enforce credit market exclusion as a punishment (since it discourages new credit by interfering with repayments to new creditors). The debtor settled with that holdout creditor in 2016. Since then, New York courts have clarified that the decision to repay some creditors but not others in and of itself does not breach the contractual clause that underpinned NML’s strategy. And that successful strategy will become increasingly difficult to pursue as enhanced CACs become more widespread. In almost all cases, litigants were not able to seize assets or disrupt ongoing restructurings (IMF, 2020).

It is possible that direct sanctions do not play a significant role because borrowers do follow procedures and norms that have come to be expected during a debt restructuring. Debtors typically ask only for the relief needed to restore sustainability with reasonable buffers, and most restructurings take place in the context of an IMF-supported program to facilitate the adjustment (and where a debt sustainability analysis informs the envelope of relief needed to restore sustainability based on the feasible adjustment). In practice, even debtors that take a fairly adversarial position with respect to their creditors follow these norms and are eager to bring the restructuring to a close. This is illustrated by the fact that if anything, restructurings tend to deliver “too little” relief (IMF, 2013; Asonuma and He, 2023). EM bond restructurings are rarely associated with significant face-value reductions, with much of the net present value relief coming from the rescheduling of debt at a moderate interest rate (significantly lower than the country would face if it were to issue new debt in the absence of a restructuring).

If countries were to deviate from these norms, e.g. ask for significant relief where it is not needed, they would likely face stronger pressures from the international community. But it is hard to test such hypothesis in the absence of counterfactuals. Even if such considerations play a role in practice, we still have the question of what costs can sustain debt relative to the choice of restructuring following established norms.

### 3. MODEL

To study the effect of uncertainty about costs of default on debt tolerance, we consider a model of a small open economy whose government borrows from competitive international lenders to frontload consumption and smooth shocks. The debt takes the form of long-term noncontingent, but defaultable, bonds. To this standard setup, we add the possibility that defaulting on the debt may carry permanent or transitory costs.

**Resources** The economy receives an endowment stream following a stochastic process with trend and cycle components

$$Y_t = \exp(z_t)\Gamma_t \quad (1)$$

with

$$\begin{aligned} z_t &= \rho z_{t-1} + \epsilon_t^z \\ \log(\Gamma_t) &= \log(\Gamma_{t-1}) + \log(g_t) \end{aligned}$$

where  $\epsilon_t^z \stackrel{iid}{\sim} \mathcal{N}(0, \sigma_z)$  and  $\log(g_t) \stackrel{iid}{\sim} \mathcal{N}(0, \sigma_g)$ . We keep the formulation for the trend  $\Gamma$  as parsimonious as possible since its only purpose is to enable possibly permanent costs of default, which would manifest as ex-post decreases in  $\Gamma$ .

We first present the model in non-stationary form. In Section 3.2, we normalize variables by  $\Gamma_t$ .

**Assets** The government borrows from international lenders in the form of a defaultable bond which promises to pay a noncontingent stream of geometrically-decaying coupons as in Leland (1998), Hatchondo and Martinez (2009), and Chatterjee and Eyigungor (2012). A bond issued in period  $t$  pays  $(1 - \rho)^{s-1}\kappa$  units of the good in period  $t + s$ , which effectively makes a one-period-old bond a perfect substitute of  $(1 - \rho)$  units of newly-issued debt. The coupon rate  $\kappa = r + \rho$ , where  $r$  is the international risk-free rate, is chosen so that the price of a bond that is expected to never default is  $q^* = 1$ .

Upon default, the government loses access to international capital markets and faces a loss of output. There is uncertainty about whether this loss of output is permanent or transitory, as well as about the length of the exclusion period.

**Government** The government is benevolent and makes choices on a sequential basis to maximize the utility of a representative household with preferences given by

$$V_t = \mathbb{E}_t \left[ \sum_{s=0}^{\infty} \beta^s \left( u(C_{t+s}) + \Gamma_{t+s}^{1-\gamma} \varepsilon_{t+s} \right) \right] = \mathbb{E}_t \left[ \sum_{s=0}^{\infty} \beta^s \Gamma_{t+s}^{1-\gamma} \left( u(c_{t+s}) + \varepsilon_{t+s} \right) \right] \quad (2)$$

where  $\mathbb{E}$  denotes the expectation operator,  $C_t$  represents the household's consumption,  $\beta$  is a discount factor, and  $\varepsilon$  is a preference shock for default or repayment,<sup>9</sup> scaled by  $\Gamma^{1-\gamma}$  to ensure that

---

<sup>9</sup>We follow Chatterjee et al. (2018) and Dvorkin et al. (2021) and introduce preference shocks for repayment and default to improve the numerical convergence of the algorithm used to solve the model. While these papers also introduce preference shocks for the borrowing decision, this is not strictly needed for convergence in our case.



its importance for the government's decisions remains constant in the face of a non-stationary endowment process.

While the government is not in default, it chooses whether to repay the debt and attains a value

$$V(B, z, \Gamma) = \mathbb{E} \left[ \max \left\{ V_R(B, z, \Gamma) + \Gamma^{1-\gamma} \epsilon_R, V_D(z, \Gamma) + \Gamma^{1-\gamma} \epsilon_D \right\} \right] \quad (3)$$

where the  $\epsilon$ 's follow a Type 1 Extreme Value distribution as in [Chatterjee et al. \(2018\)](#), yielding closed forms for the value function in the normalized formulation below

**Debt issuances** If the government chooses to repay the debt, it can access capital markets and issue new debt  $B'$ , so that

$$\begin{aligned} V_R(B, z, \Gamma) &= \max_{B'} u(C) + \beta \mathbb{E} [V(B', z', \Gamma g') \mid z] \\ \text{subject to } C + \kappa B &= Y(z, \Gamma) + q(B', z, \Gamma)(B' - (1 - \rho)B) \end{aligned} \quad (4)$$

**Default decision** If the government chooses to default, it loses access to international capital markets, which it then recovers with a constant hazard  $\psi$ . While it is excluded, the borrowing economy suffers a loss of output  $\Delta$ . Upon default, a shock  $k \in \{T, P\}$  determines whether this loss of output is transitory or permanent, which happen with probability  $p$  and  $1 - p$ , respectively. We therefore maintain the usual assumption that the resumption of payments and the renewed access to international markets happen simultaneously.<sup>10</sup> Our departure from the standard model only involves that the output costs of default do not necessarily dissipate when market access is restored.

Regardless of their persistence, the output costs of default are always of the same size and reduce output by a factor of  $\Delta$ . This linearity assumption allows to cleanly separate transitory costs affecting  $z$  from permanent ones which impact  $\Gamma$ . In other words, if output in default is  $Y^D = \exp(z)\Gamma(1 - \Delta)$ , we can express it as a reduction in  $z$  when the costs are transitory and a

---

Rather, the preference shocks for repayment are sufficient to replace the two-tiered income process used early in the literature of sovereign default with long-term debt (see for example [Chatterjee and Eyigungor, 2012](#)). As is also common, we keep the variance of the preference shocks small to ensure that they do not modify the quantitative properties of the model solution. Plausible stories to microfound a more meaningful inclusion of such shocks include shifting political preferences, developments in domestic debt markets or in the banking system, or changes in the external borrowing terms faced by private firms. [Fourakis \(2023\)](#) uses preference shocks with a larger variance to obscure the actions taken by two possible 'types' of the government and thus enable a signalling dimension to government borrowing, default, and renegotiation.

<sup>10</sup>See [Mihalache \(2020\)](#) for a case in which market access is restored after debt service payments have resumed.

reduction in  $\Gamma$  when they are permanent. Below we show how this separation helps us save a state variable.

We assume that all debt is destroyed after a default for tractability and to more readily compare with other results in the literature.<sup>11</sup> The expected value of default is

$$V_D(z, \Gamma) = pV_D^T(z, \Gamma) + (1 - p)V_D^P(z, \Gamma(1 - \Delta)) \quad (5)$$

and

$$V_D^k(z, \Gamma) = u(Y(z, \Gamma)(1 - 1_{(k=T)}\Delta)) + \beta\mathbb{E} [\psi V(0, z', \Gamma g') + (1 - \psi)V_D^k(z', \Gamma g') \mid z] \quad (6)$$

which illustrate how the permanent component of output  $\Gamma$  is reduced (in equation 5) when the costs are permanent, while output itself is reduced (in equation 6), for the duration of the exclusion spell, when costs are transitory.

**Lenders** Bonds issued by the government are purchased by deep-pocketed, risk-neutral foreign investors who equate the expected return of the debt to their cost of funds  $r$ , yielding a debt price

$$q(B', z, \Gamma) = \frac{1}{1 + r}\mathbb{E} [(1 - \mathcal{P}(B', z', \Gamma g'))(\kappa + (1 - \rho)q(B'', z', \Gamma g')) \mid z] \quad (7)$$

where  $B'$  and  $\mathcal{P}$  denote the government's issuance and default policy expected by lenders. We implicitly assume these are private investors.<sup>12</sup>

### 3.1 Equilibrium

We define an equilibrium of the economy in its original non-stationary form before proceeding to normalizing the model to compute a solution.

**Definition.** A Markov-Perfect equilibrium consists of a set of value functions  $V$ ,  $V_R$ ,  $V_D$ ,  $V_D^T$  and  $V_D^P$ , a default probability  $\mathcal{P}$ , a borrowing rule  $B'$ , and a bond price function  $q$ , such that

<sup>11</sup>In practice, restructurings tend to involve substantial recovery rates. Meyer, Reinhart, and Trebesch (2022) estimate an average recovery above 50 in a long historical sample. Adding a recovery rate of that magnitude would reduce the benefits of restructuring, and hence help support more debt in equilibrium. But a positive recovery rate should not interact with the forces we highlight in this model.

<sup>12</sup>In principle, the model could apply to any risk-neutral investor. The processes for restructuring private and official bilateral debt are typically different, but to the extent that our model assumes no recovery after a default that would not make a difference. There likely are other factors influencing the decision to lend in the case of official creditors that are not captured by our model. For recent studies exploring the interaction of private and different types of official sovereign borrowing, see Boz (2011), Hatchondo, Martinez, and Onder (2017), Arellano and Barreto (2023) and Roldán and Sosa-Padilla (2023).

- i. Given the price of debt, the value functions satisfy functional equations (3)-(6).
- ii. Given the default probability and borrowing rules, the price of debt satisfies the creditors' Euler equation (7).
- iii. Given the price of debt, the default probability and borrowing rules correspond to the policy functions solving (3) and (4).<sup>13</sup>

### 3.2 Normalization

We detrend variables by  $\Gamma_t$  and denote normalized values with lowercase. For example,  $y_t = Y_t/\Gamma_t = \exp(z_t)$ . We also specialize the utility function to a CRRA form, and obtain

$$u(C) = \frac{C^{1-\gamma}}{1-\gamma} = \Gamma^{1-\gamma} \frac{(C/\Gamma)^{1-\gamma}}{1-\gamma} = \Gamma^{1-\gamma} u(c)$$

which implies a normalization constant  $\Gamma_t^{1-\gamma}$  for the value functions, as laid out in more detail in Appendix B.

Normalizing variables by  $\Gamma$  leads to a formulation with state variables  $(b, z, g)$ . At the beginning of a period, the government faces the choice of default and repayment (3), where we guess-and-verify (along the lines of Appendix B) that  $V(B, z, \Gamma) = \Gamma^{1-\gamma} v(b, z, g)$  and multiply both sides by  $\Gamma^{\gamma-1}$  to obtain

$$v(b, z, g) = \mathbb{E} \left[ \max \left\{ v_R(b, z, g) + \epsilon_R, v_D(z, g) + \epsilon_D \right\} \right] \quad (8)$$

As in Chatterjee et al. (2018), the distributional assumption for the  $\epsilon$ 's implies closed forms for the value function  $v(b, z, g)$  and the (ex-post) default probability  $\mathcal{P}(b, z, g)$

$$\begin{aligned} v(b, z, g) &= \chi \log \left( \exp(v_D(z, g)/\chi) + \exp(v_R(b, z, g)/\chi) \right) \\ \mathcal{P}(b, z, g) &= \frac{\exp(v_D(z, g)/\chi)}{\exp(v_D(z, g)/\chi) + \exp(v_R(b, z, g)/\chi)} \end{aligned} \quad (9)$$

where the  $\epsilon$ 's are distributed Extreme Value Type 1 with scale parameter  $\chi$ .

Casting the equations describing the values for repayment and default in stationary form turns out to be less trivial. For (4), notice that normalizing by the current  $\Gamma$  creates a difference between the control variable  $b' = B'/\Gamma$  which appears in the budget constraint and the following

---

<sup>13</sup>Aguiar and Amador (2020) show that in an Eaton-Gersovitz model with long-term debt, there may be multiple Markov Perfect equilibria. We rule out this possibility by focusing on the equilibrium that is the limit of the equilibrium of the finite-horizon economy.

period's state  $b' = B'/\Gamma'$  which appears in the continuation value function. To reserve the symbol  $b'$  for the second meaning, we will denote the current period's issuance  $B'/\Gamma$  as  $x$ . Multiplying both sides of (4) by  $\Gamma^{\gamma-1}$  yields

$$\begin{aligned}\Gamma^{\gamma-1}V_R(B, z, \Gamma) &= \max_{B'} \Gamma^{\gamma-1}u(C) + \beta\mathbb{E} \left[ \Gamma^{\gamma-1} \left( \frac{g'}{g} \right)^{\gamma-1} V(B', z', \Gamma g') \mid z \right] \\ v_R(b, z, g) &= \max_{b'} u(c) + \beta\mathbb{E} \left[ (g')^{1-\gamma} v(b', z', g') \mid z \right]\end{aligned}\quad (10)$$

where the maximization is subject to the budget constraint

$$\begin{aligned}\frac{C + \kappa B}{\Gamma} &= \frac{Y(z, \Gamma) + q(B', z, \Gamma)(B' - (1 - \rho)B)}{\Gamma} \\ c + \kappa b &= y(z) + q(x, z, g)(x - (1 - \rho)b)\end{aligned}$$

where we also used that  $Y(z, \Gamma)/\Gamma = \exp(z)$  and can drop the dependence on the non-stationary part entirely. Also note that it is equivalent to choose  $b'$  or  $x$  as long as the relationship between them,  $b' = x/g'$ , is imposed where appropriate.

Notice that, as usual in non-stationary models, the stochastic discount factor now becomes  $\beta(g')^{1-\gamma}$ . Trend growth and impatience impact borrowing decision in the same way. We choose for simplicity a formulation without drift in the stochastic process for  $g$ , effectively loading mean growth onto the discount factor.

Finally, to cast the value of default in stationary form, notice that the value of  $\Gamma$ , by which we are normalizing, varies when the costs turn out to be permanent. Starting from (5) and multiplying by  $\Gamma^{\gamma-1}$ , we have

$$\begin{aligned}\Gamma^{\gamma-1}V_D(z, \Gamma) &= p\Gamma^{\gamma-1}V_D^T(z, \Gamma) + (1 - p)\Gamma^{\gamma-1} \left( \frac{\Gamma(1 - \Delta)}{\Gamma(1 - \Delta)} \right)^{\gamma-1} V_D^P(z, \Gamma(1 - \Delta)) \\ v_D(z, g) &= p v_D^T(z, g) + (1 - p)(1 - \Delta)^{1-\gamma} v_D^P(z, g(1 - \Delta))\end{aligned}\quad (11)$$

and the conditional value of default follows similarly

$$v_D^k(z, g) = u(y(z)(1 - 1_{(k=T)}\Delta)) + \beta\mathbb{E} \left[ (g')^{1-\gamma} (\psi v(0, z', g') + (1 - \psi)v_D^k(z', g')) \mid z \right] \quad (12)$$

The price of debt (7) is relatively unaffected by the normalization but features the distinction between  $x = B'/\Gamma$ , the debt chosen in the current period, and  $b' = B'/\Gamma'$ , the debt outstanding at the beginning of the following period

$$q(x, z, g) = \frac{1}{1 + r} \mathbb{E} \left[ (1 - \mathcal{P}(x/g', z', g')) (\kappa + (1 - \rho)q(x', z', g')) \mid z \right] \quad (13)$$

Notice that because  $g$  is *iid*, the costs of default can be applied to  $g$  or  $z$  depending on their permanence, and the factor  $\Delta$  is independent of both  $z$  and  $g$ , there is no need to carry  $g$  as a state variable in any of the value or price functions. Conversely, autocorrelated  $g$  shocks would require conditioning all expectations on the current  $g$ , and more general costs (for example, strictly convex costs as a function of  $Y$ ), would require a renormalization factor in (11) with an explicit dependence on  $g$  (and possibly  $z$ ). With these assumptions, we drop redundant dependencies on  $g$  in what follows.

Although advantageous, our formulation with linear costs is less flexible than the strictly convex costs sometimes used in the literature. Convex costs make it ‘cheaper’ to default when income is low and therefore induce more countercyclical default probabilities and spreads. For the same reason, convex costs make ex-post default more dependent on the following period’s income shocks; in other words they induce more uncertainty in default probabilities. This feature can be seen as similar to the uncertainty introduced by the preference shocks and manifested by the ex-post default probability  $\mathcal{P}$ . However, the variance of our preference shocks is chosen to be small, to ensure that  $\mathcal{P}$  mostly takes values close to 0 and 1.

### 3.3 Model with robust preferences

There are five shocks in the model,  $\epsilon^z$  and  $g$  for the endowment process, the preference shock  $\epsilon_R - \epsilon_D$  for repayment, the reentry to international markets after a default, and whether the costs of defaulting are permanent or transitory. We assume that, while the true stochastic processes governing these shocks are unknown, all agents in the economy take the descriptions above to be approximating models. While foreign creditors trust the approximating model and use it to compute the expectation in (7), the government does not. The government is instead concerned about potential misspecification of (some of) these processes and seeks robust decision rules.

We represent such doubts and objectives with *multiplier preferences* (Hansen and Sargent, 2001) which capture ambiguity aversion with a single parameter. In this framework, robust decision rules emerge from confronting the decision maker with an ‘evil agent’ alter-ego. This fictitious, auxiliary agent chooses a worst-case model to minimize the decision maker’s utility for each possible choice of action. Critically, the evil agent faces an entropy cost  $\frac{1}{\theta}$  which only allows it to pick distortions from the approximating model (worst-case models) which are difficult to statistically distinguish from it. When the marginal cost of this relative entropy is infinite (when  $\theta \rightarrow 0$ ), we recover the standard expected-utility decision maker. In what follows, we refer to the inverse of the marginal cost of relative entropy,  $\theta$ , as the robustness parameter.

Moreover, the government potentially places different degrees of trust in its model for the endowment process, which is observed continuously, and in its model for the default costs, which are observed infrequently.<sup>14</sup> We therefore denote as  $\theta_s$  and  $\theta_c$  the two robustness parameters for evaluating expectations involving  $(z', g')$  and  $k$ , respectively. Finally, we assume that the government does not doubt the stochastic process for reentry to markets after a default. While it is difficult to empirically determine whether output eventually catches up to its pre-default trend, exclusion periods are generally better understood (see [Cruces and Trebesch, 2013](#)).<sup>15</sup>

With multiplier preferences, the government's problems and value functions are<sup>16</sup>

$$\begin{aligned} v_R(b, z) &= \max_x u(c) + \beta \frac{1}{-\theta_s} \log (\mathbb{E} [\exp (-\theta_s (g')^{1-\gamma} v(x/g', z')) \mid z]) \\ &\text{subject to } c + \kappa b = y(z) + q(x, z)(x - (1 - \rho)b) \end{aligned} \quad (14)$$

$$v_D(z) = \frac{1}{-\theta_c} \log (p \exp (-\theta_c v_D^T(z)) + (1 - p) \exp (-\theta_c (1 - \Delta)^{1-\gamma} v_D^P(z))) \quad (15)$$

and

$$v_D^k(z) = u(y(z)(1 - 1_{(k=T)}\Delta)) + \beta \frac{1}{-\theta_s} \log (\mathbb{E} [\exp (-\theta_s (g')^{1-\gamma} (\psi v(0, z') + (1 - \psi) v_D^k(z')) \mid z]) \quad (16)$$

It is easy to show that this formulation with robustness converges back to the standard expected utility case as  $(\theta_s, \theta_c) \rightarrow 0$ . Appendix D discusses the numerical solution of the model in this general case.

In general, under certain conditions it can be shown that multiplier preferences are mathematically equivalent to risk-sensitive Epstein-Zin preferences ([Strzalecki, 2011](#)). While risk-sensitive preferences describe an agent with certain attitudes when confronted to risk, robustness links such attitudes to the agent's reluctance to fully trust the model underpinning expectations. As such, robustness becomes a natural assumption in the context of uncertainty about the nature of default costs, a topic on which research is still ongoing. Finally, the flexibility of robustness relative to general risk-sensitive preferences allows us to focus on specific sources of ambiguity by distinguishing the risk-attitudes towards uncertainty in the continuously-observed and better-understood endowment process, represented by  $\theta_s$ , and towards uncertainty in the persistence

<sup>14</sup>We assume in all cases that the remaining shocks are not subject to potential misspecification. The preference shocks  $(\epsilon_R, \epsilon_D)$  are only introduced for numerical performance and with a low variance.

<sup>15</sup>[Fourakis \(2023\)](#) provides a model of signalling to understand the response of interest rates after market access has been recovered.

<sup>16</sup>For more details about how multiplier preferences yield a Bellman equation of the form of (14), see Appendix C.

of the output costs of default which are observed much more infrequently, represented by  $\theta_c$ .<sup>17</sup> Below, we will starkly assume that the government trusts the model for the endowment shocks (effectively setting  $\theta_s = 0$ ) but doubts the model for default costs, represented by  $\theta_c > 0$ .

As a benchmark, we maintain the assumption of risk-neutral expected-utility preferences for the creditors. In other words, while everyone in the model shares the same approximating or baseline model, only the government mistrusts (particular aspects of) it. The government understands that creditors fully trust the baseline model and may sometimes note with relief that borrowing costs would be higher if the creditors shared its concerns about misspecification.<sup>18</sup>

### 3.3.1 Worst-case models

An appealing feature of robustness relative to risk-sensitive preferences is that it is possible to recover the belief distortions underpinning particular actions, the ‘worst-case model.’ Let the *distorted expectation* of random variable  $X$  at information set  $\mathcal{F}$  be the objective expectation of the product of  $X$  with a likelihood ratio

$$\tilde{\mathbb{E}}[X | \mathcal{F}] = \mathbb{E} \left[ \frac{\exp(-\theta v)}{\mathbb{E}[\exp(-\theta v) | \mathcal{F}]} X \mid \mathcal{F} \right] \quad (17)$$

where  $v$  is the government’s value function at that information set.<sup>19</sup>

As compared to an expectation taken with the objective probability measure (the approximating or baseline model), the distorted expectation magnifies the likelihood of states for which the government’s utility, measured by the value function  $v$ , is low. This procedure results in endogenously pessimistic beliefs which sustain robust decision-making. The role of the robustness parameter  $\theta$  can also be clearly understood from (17). A larger value for  $\theta$  means stronger misspecification concerns on the part of the government, and consequently a more distorted expectation. Conversely, as  $\theta \rightarrow 0$ , misspecification concerns vanish and the distorted expectation converges back to the objective one.

---

<sup>17</sup>Distinguishing two robustness parameters in this way can also be viewed as an extreme form of robustness to “structured uncertainty” (Hansen and Sargent, 2015) when the decision maker’s misspecification concerns focus on particular features of its beliefs.

<sup>18</sup>The fact that worst-case models are endogenous makes it difficult to exploit gains from trade arising from different beliefs within a certain equilibrium. Manipulating the counterpart’s worst-case model is a central theme in Roch and Roldán (2023), where a government designs GDP-linked bonds to borrow from robust creditors, or in Karantounias (2023), where a policymaker manages the private sector’s expectations in order to relax forward-looking constraints.

<sup>19</sup>Readers who strongly favor risk-sensitive preferences can interpret our worst-case models as the government’s risk-neutral measure.

### 3.3.2 Model detection-error probabilities

As is common in the robustness literature, we compute model detection-error probabilities (DEP) to help inform and discipline our choice of  $\theta_s$  and  $\theta_c$ .<sup>20</sup> When the government follows robust decision rules, it does so by computing an auxiliary worst-case model. The DEP captures the probability that an agent, with a limited amount of data, mistakes data generated by one of the models as coming from the other one. A DEP of 50% means that the baseline and worst-case models are observationally equivalent while a DEP of 0 means that an agent can perfectly distinguish both models. A high DEP suggests that the amount of misspecification implicit in the distorted model is plausible and validates the decision maker's desire for robustness.

We compute model detection-error probabilities as the probability of misclassifying a sample from the approximating model  $A$  as coming from the (endogenous) worst-case model  $W$  and viceversa. In each case we compute the likelihood  $L(x | M)$  of data  $x$  being generated by model  $M$  to classify the sample as coming from the model which produces the maximum likelihood. Finally, the detection-error probability is the average of the probability of misclassification when the true model is  $W$  and when it is  $A$ .<sup>21</sup>

$$DEP = \frac{\mathbb{P}(L(x | A) > L(x | W) | W) + \mathbb{P}(L(x | W) > L(x | A) | A)}{2} \quad (18)$$

We note that it seems natural that distortions to the stochastic process governing the type of default costs, which as we show below can have a large impact on the government's decisions, can be difficult to detect as this shock is only observed infrequently. On the other hand, distortions of similar magnitude applied to the continuously-observed shocks governing the endowment would be much more salient. This asymmetry motivates our approach of assuming different robustness parameters for these two forms of uncertainty, while recognizing that a fuller approach in which the government's evil agent has, for example, an entropy budget to allocate to both types of shocks, so as to analyze which shock is distorted more, is left outside the scope of this paper.

---

<sup>20</sup>We leave the model description general even though we only consider positive values for  $\theta_c$ .

<sup>21</sup>We implement these calculations below on 60-year samples (which is more than the length of quarterly GDP data for most EM countries), after a 500-period burn-in to ensure draws from the ergodic distribution of each model, over 2000 repetitions.



## 4. CALIBRATION

### 4.1 *Output trajectories around restructuring episodes*

We summarize the evidence on output costs of restructuring by computing deviations of output around debt restructuring episodes from a pre-restructuring trend in the [Asonuma and Trebesch \(2016\)](#) dataset.<sup>22</sup> We purposefully abstract from causality. Instead, our approach is to parametrize the model to ensure that it generates the same correlations we find in the data. This allows us to indirectly infer the underlying causal parameters determining the size and persistence of default costs.

We consider a panel of emerging market economies (EMs) which have had a sovereign restructuring event in 1990-2020 for which data on external debt, standing in international capital markets, GDP, and spreads are all available for at least 10 years previous to the default event. For each restructuring episode occurring at time  $t$  in country  $i$ , we construct a pre-restructuring trend for output by regressing log output in years  $t - j$ , for  $1 \leq j \leq 6$ , against a linear function of time

$$\log Y_{i,t-j} = \alpha_{it} + \beta_{it}(t - j) + \epsilon_{ij} \quad (19)$$

We then use this pre-restructuring trend to obtain fitted values  $\hat{Y}_{i,t+k}$  and compare realized output  $Y_{i,t+k}$  against this trend at horizon  $k$ . We focus on horizons  $k = 1$  and  $5$  and take the median across all episodes. We complement these calculations with values for external debt (as share of GDP) and spreads, taken as averages between  $t - 10$  and  $t - 5$ .

Table 1 summarizes our results, which we later use as calibration targets for the model presented in Section 3. The output deviation is sensitive to how the trend is defined, as the loss may be spread over many years and some of the decline relative to trend may take place in the run-up to the restructuring. We are therefore careful to estimate the same statistics in the model-generated data. We recover the structural causal cost-of-default parameters by matching the output deviations, while the overall output trajectory can be used for validation of the model. Finally, as all debt in the model is external, we compute a target for the external-debt-to-GDP ratio.<sup>23</sup>

---

<sup>22</sup>We consider all restructuring episodes in their dataset, including those that take place preemptively to avoid a default.

<sup>23</sup>This number is lower than the headline debt-to-GDP ratio at which most countries tend to experience sovereign debt defaults.

TABLE 1: CALIBRATION TARGETS

Description	Calculation	Value
Output deviation, 1-year horizon, %	$Y_{t+1}/\hat{Y}_{t+1} - 1$	8.27
Output deviation, 5-year horizon, %	$Y_{t+5}/\hat{Y}_{t+5} - 1$	7.6
Average external debt-to-GDP ratio, %	$\overline{B_{t-j}/Y_{t-j}}$	23.4
Average spread, bps	$\overline{Spread_{t-j}}$	793

#### 4.2 Baseline calibration with robustness

We parametrize our model at a quarterly frequency. We externally set most parameters (the sovereign’s risk aversion  $\gamma$ , the risk-free interest rate  $r$ , the duration of debt  $\rho$  and the reentry probability  $\psi$ ) to standard values in models of sovereign default. We choose a small value for the preference shock scale parameter  $\chi$  to minimize the impact of this numerical device on the equilibrium.

We estimate the endowment process parameters  $(\rho_z, \sigma_z, \sigma_g)$  for each country in our sample in the period before the first default, starting in 1960, and take averages across the episodes. In each case, we run the log of output through an HP filter and estimate an AR(1) process on the cycle component to obtain  $(\rho_z, \sigma_z)$  and retrieve  $\sigma_g$  as the standard deviation of the residuals of the trend component against a linear time trend.

Finally, we assume that the government places full trust in its approximating model for the income process, so  $\theta_s = 0$ . This leaves us the sovereign’s discount factor  $\beta$ , the level of default costs  $\Delta$ , the probability of a transitory cost  $p$  (under the approximating model), and the robustness parameter  $\theta_c$ . We set these parameters to match the moments from Table 1: the local projections for output at 1 and 5 years after a default event, relative to a pre-default trend, the average level of spreads and the average debt-to-GDP ratio while in repayment. Table 2 summarizes our parametrization.

To assess the match to the moments from Table 1, we generate simulated data from the model<sup>24</sup> and filter it through the same procedure used for the sample of EM default episodes. The calibrated model matches those patterns closely, as shown in Table 3. We note that our structural estimate for the costs of default ( $\Delta = 4.11\%$ ) falls well within the range of empirical estimates of the causal effect, and quite close to the value found by [Asonuma et al. \(2023\)](#) which controls for a number

<sup>24</sup>We take 2000 simulations of 250 years in each case.

TABLE 2: BASELINE PARAMETER VALUES

Externally chosen		
	Parameter	Value
Sovereign's risk aversion	$\gamma$	2
Preference shock scale parameter	$\chi$	0.01
Risk-free interest rate	$r$	0.01
Robustness parameter: income shocks	$\theta_s$	0
Duration of debt	$\rho$	0.05
Reentry probability	$\psi$	0.0385
Income autocorrelation coefficient	$\rho_z$	0.9256
Standard deviation of $z_t$	$\sigma_z$	0.0231
Standard deviation of $g_t$	$\sigma_g$	0.0211
Internally calibrated		
	Parameter	Value
Sovereign's discount factor	$\beta$	0.902
Default cost	$\Delta$	0.0411
Probability of transitory shock	$p$	0.339
Robustness parameter: default costs	$\theta_c$	7.6

of variables as well as the endogeneity of the choice of restructuring.

TABLE 3: MODEL FIT

	Data	Model
Output deviation, 1-year horizon, %	8.27	9.75
Output deviation, 5-year horizon, %	7.6	7.99
Average external debt-to-GDP ratio, %	23.4	21.3
Average spread, bps	793	813

Figure 2 plots the entire trajectories for output around default events in the data, with a shaded area indicating the interquartile range, as well as in the model. In the model we take the average simulation path conditional on default at year 0 and no defaults in the previous 5 years to replicate the selection done in the data. The model-generated dynamics match the data counterparts well,

especially taking into account that the (annual) data reflects some time-averaging relative to our quarterly model. Both data and model exhibit a slight acceleration of output in the run-up to the default (between years -5 and -2) followed by a series of negative shocks starting around 1.5 years before the default. Both model and data then show a slow recovery in the years following the episode.

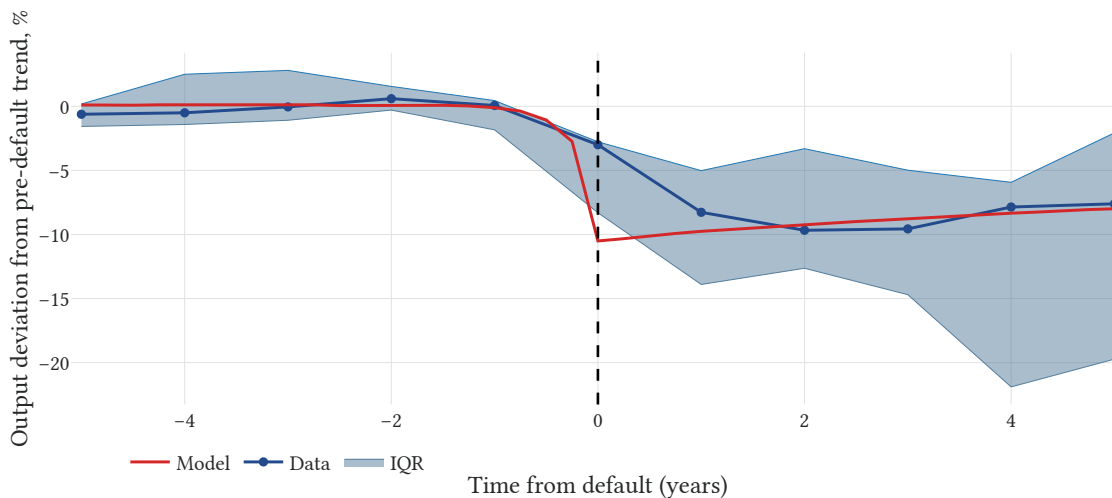


FIGURE 2: OUTPUT TRAJECTORIES AROUND DEFAULTS

## 5. QUANTITATIVE RESULTS

We begin by presenting the moments targeted in the calibration for our benchmark model, along with those same moments for a version without robustness (in which we simply  $\theta_c = 0$ ) and for a version without robustness, but where we recalibrate the remaining three parameters. Table 4 summarizes the results

Here table with different calibrations

TABLE 4: TARGETED MOMENTS

Notice that, because robustness has no effect if there is no underlying uncertainty, we do not solve the model introducing only robustness.

Comments on how without robustness you get lower debt levels and too high actual costs when calibrating without  $\theta_c$

Table 5 provides a comparison of the general performance of these models along typical moments in the sovereign debt literature.

Here table with other moments

TABLE 5: OTHER ERGODIC MOMENTS

### 5.1 Actual and perceived persistence of default costs

Figure 3 shows simulation results at the baseline parametrization, varying the probability  $p$  that default costs are transitory and the robustness parameter  $\theta_c$  affecting this particular source of uncertainty. We solve and simulate the economy for each combination of  $(p, \theta_c)$  and report the average debt-to-GDP ratio  $B/Y$  and the default frequency, both in yearly terms.

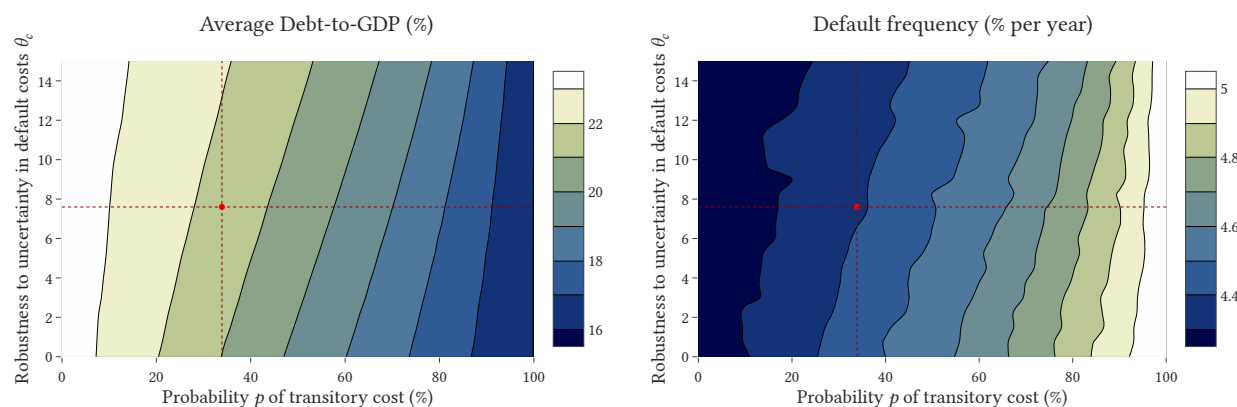


FIGURE 3: DEBT TOLERANCE VARYING  $p$  AND  $\theta_c$

Note: Red dots denote the baseline calibration.

When the shock is more likely to be permanent, or the government has greater misspecification concerns, the left panel of Figure 3 shows that the economy sustains larger levels of debt. Robustness has a mechanically larger impact when the underlying uncertainty about the permanence of costs is greater (i.e. when  $p$  is closer to 50%). Our calibrated value for  $p$  is about 35% which is squarely in the range where robustness has more quantitative bite. Compared to a parametrization in which the government is convinced that the cost is transitory ( $p = 1$ , the standard case in the literature), the average level of debt increases by about a quarter. Making the cost permanent for sure would result in a further increase in average debt of about 15% from the baseline calibration level.

The right panel reveals that the default frequency is less sensitive to either of these parameters, and relatively more insensitive to the degree of robustness.<sup>25</sup> Shifting the cost from certainly permanent to certainly transitory only increases the default frequency by about 17% (or

<sup>25</sup>At the calibrated level of  $p$ , shifting  $\theta_c$  moves debt levels but leaves the default frequency relatively unchanged.

about 0.3p.p.). Thus, most of the increase in perceived costs of default (both through actual persistence and robustness) translates into additional borrowing. The debtor government uses the extra ‘credibility’ afforded by the higher default costs to sustain higher levels of indebtedness, rather than keeping debt the same and facing the costs of default less frequently.

## 5.2 Decomposition of output costs of default

The calibrated model matches the output trajectory estimated from the data (Figure 2). In the model, we can decompose the output deviations according to<sup>26</sup>

$$\log Y_t - \log \hat{Y}_t = z_t + \log \Gamma_t + \log(1 - \Delta)1_{(D_t=1)} - \log \hat{Y}_t \quad (20)$$

where  $1_{(D_t=1)}$  refers to whether costs of default are active in period  $t$ , which could result from the economy being excluded in the current period or from a permanent cost of default in the current episode. We further normalize  $Y_t$ ,  $\hat{Y}_t$ , and  $\Gamma_t$  to the trend level at the start of each episode such that  $\Gamma_{t-21} = 1$ .

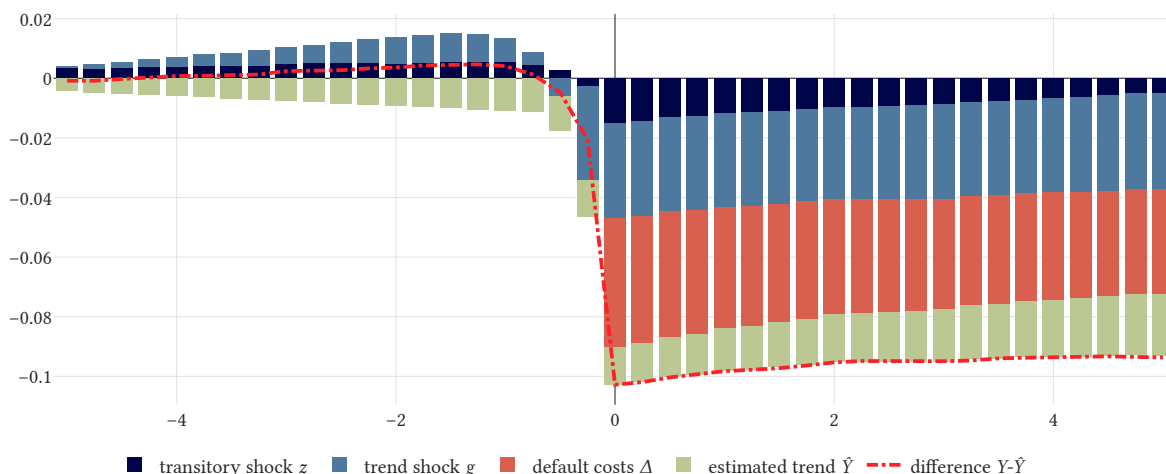


FIGURE 4: DECOMPOSITION OF OUTPUT DEVIATIONS FROM TREND

Figure 4 plots the decomposition and reveals that actual, ‘causal’ default costs account for about 35% of the output deviation from trend. Another 40% comes from underlying shocks which

<sup>26</sup>To obtain an exact decomposition in this case, we measure the output deviation from trend as log differences, which leads to slightly different headline numbers than in the calibration and Figure 2, where we measured such deviations as ratios.

tend to be negative at the time of default. Finally, about 25% of the deviation comes from the fact that output is growing (more than usual) in the pre-default period, leading to an upward trend later on. By construction, our sampling includes a restructuring at  $t = 0$  but not before, which selects a relatively strong pre-crisis trend. The level shocks  $g_t$  then turn negative as we approach the time of the default event  $t = 0$ . These considerations explain why it would be hard for output to recover to the pre-crisis trend even if the default costs were to completely disappear, and how simple before-and-after growth comparisons can be misleading.

### 5.3 The effect of robustness

make sure this section works with minimal repetition

We focus on how robustness amplifies the perceived probability of a permanent cost of default. Because this variable  $k \in \{P, T\}$  is only realized after a default has taken place, the appropriate value function to use in (17) is  $v_D^k$ ,<sup>27</sup> with the renormalization factor  $(1 - \Delta)^{1-\gamma}$  in case the cost is permanent. Letting  $\tilde{p}(z)$  denote the distorted probability that the cost is transitory (i.e.  $k = T$ ) at state  $z$ ,

$$\tilde{p}(z) = \tilde{\mathbb{E}}[1_{(k=T)} \mid z] = \mathbb{E} \left[ \frac{\exp(-\theta v_D^k(z)(1 - 1_{(k=P)}\Delta)^{1-\gamma})}{\mathbb{E} [\exp(-\theta v_D^k(z)(1 - 1_{(k=P)}\Delta)^{1-\gamma})]} 1_{(k=T)} \mid z \right]$$

Figure 5 shows that the government's misspecification concerns make it act as if the probability of a permanent cost of default was larger by a factor of about a quarter. Moreover, the distortion is larger when current income is lower, as a higher likelihood of default naturally makes the government more concerned about misspecification of the consequences of default. This effect is the key to the differential effect of robustness  $\theta_c$  and the actual probability of a permanent cost  $1 - p$ : the worst-case probability of a permanent cost increases more in low states, which is where default is more likely. As (perceived) costs become relatively larger in low states, the level of debt at which default happens becomes less dependent on the current income state  $z$ . This makes it easier to forecast whether a certain level of debt will be repaid in the future. As a consequence, when default costs increase via robustness, the government is more able to 'extract' the increase in debt tolerance while keeping the default frequency relatively unchanged. When they increase via the actual probability  $p$ , the government chooses a lower default frequency as well as a higher debt level (Figure 3).

Figure 6 illustrates the relationship between borrowing terms faced by the government and

---

<sup>27</sup>In other words, the value function at the information set corresponding to the fact that a default has taken place (and the current state is  $z$ ) is  $v_D^k(z)$

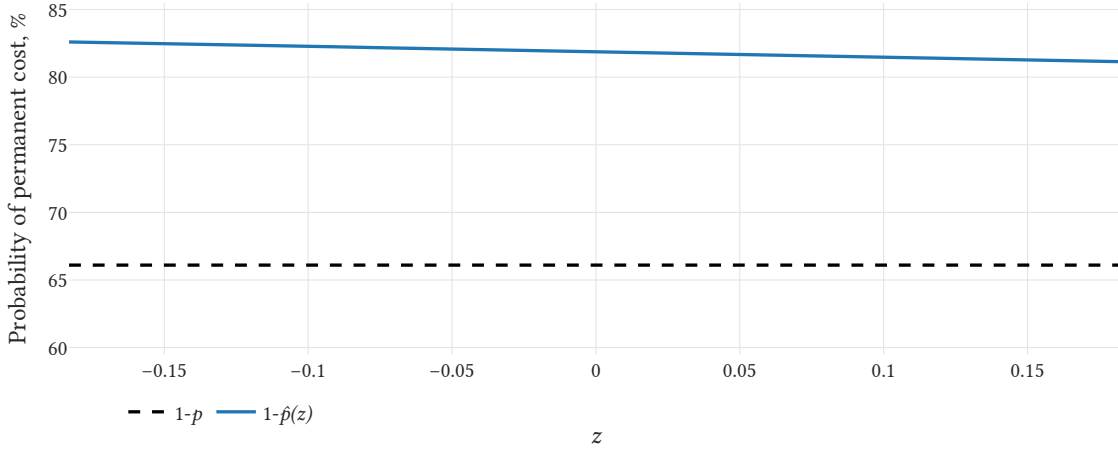


FIGURE 5: WORST-CASE PROBABILITY OF A TRANSITORY COST,  $\tilde{p}$

the costs of default, actual and perceived, it faces. For a moderate level of debt (but in the region where spreads start to increase), spreads decrease when the shock is more likely to be permanent. They also decrease when the government has stronger misspecification concerns. As before, the power of robustness is dampened when the government is more certain about the nature of costs (i.e. when  $p$  is closer to 0 or 1). The response of spreads becomes very non-linear as we approach high values of  $p$  (especially at low values for  $\theta_c$ ). At the chosen debt level (19% of mean income), the default probability would be almost 1 at such parameter values. Figure 7 recasts this same information in terms of the debt issuance level at which spreads become very large, which is perhaps easier to read in terms of debt tolerance.

#### 5.4 Model detection-error probabilities

Figure 8 plots the model detection-error probability for different values of the objective probability  $p$  of a transitory cost, as a function of the choice of the robustness parameter  $\theta_c$ . Typically, in the robust control literature, acceptable values for DEP are of 20% or above (Barillas et al., 2009). Our choices of  $\theta_c$  stay well within this constraint suggesting we rely on reasonable amounts of probability distortions.

For a given baseline probability  $p$  of a permanent shock, increasing robustness leads to higher levels of debt sustained in equilibrium. The government sustains these higher levels of debt because the worst-case model arising from the government's robustness concerns features a larger probability of permanent costs (Figure 3). This feature also makes it more easily distinguishable



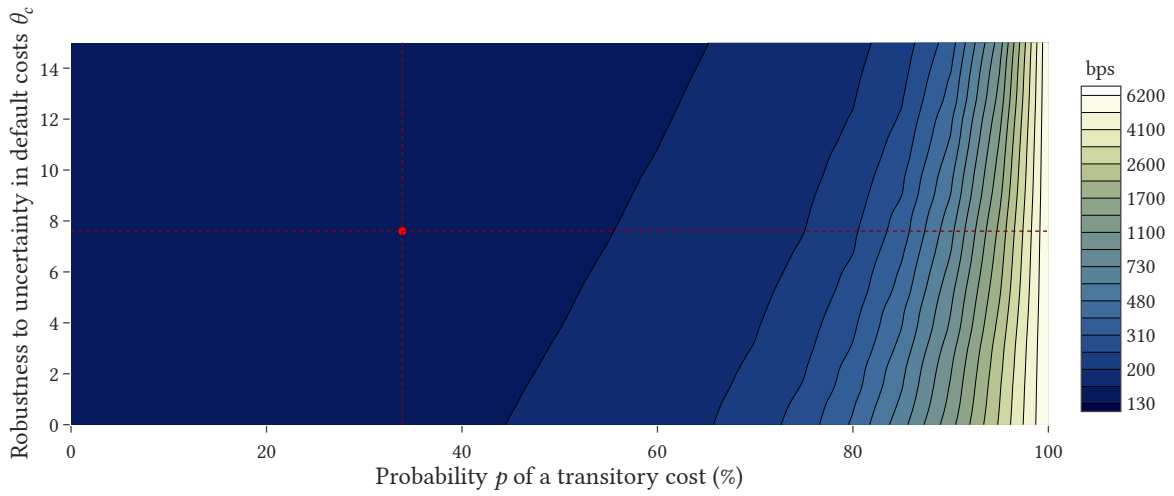


FIGURE 6: SPREADS ON GOVERNMENT DEBT

*Note:* spreads computed at mean income and debt at 19% of income in each case, red dots denote the baseline calibration

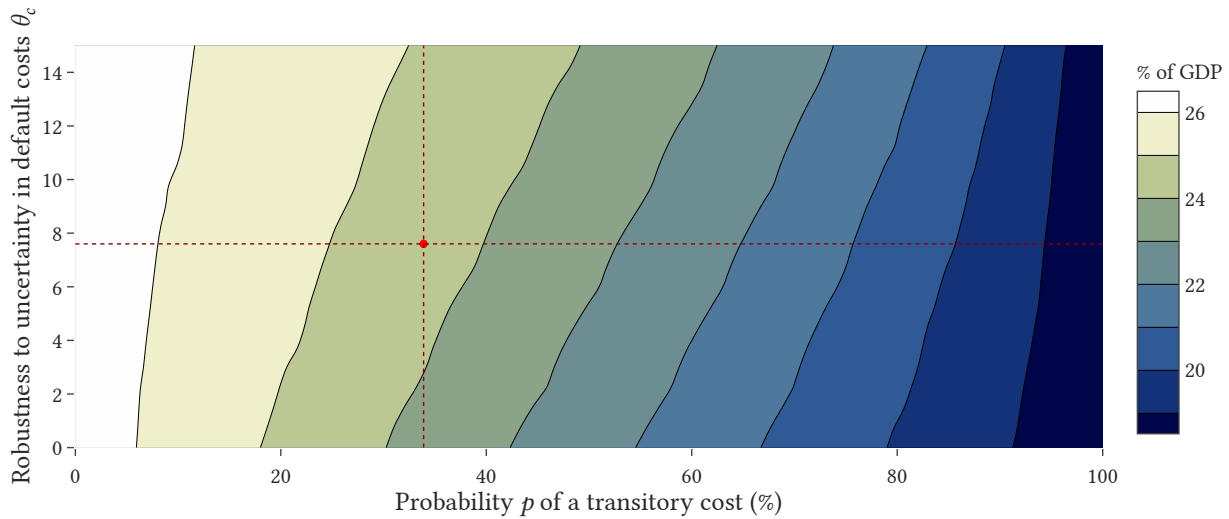


FIGURE 7: DEBT AT WHICH SPREADS CROSS 1000BPS

*Note:* Red dots denote the baseline calibration.

from the baseline model. Indeed, Figure 8 shows that the DEP decreases with the robustness parameter  $\theta_c$ . Moreover, as  $p$  grows towards 1, there is more scope to distort beliefs in a pessimistic way. The more the cost looks transitory, the more there is to fear that it is permanent. This leads to lower values for the DEP when  $p$  is large, holding robustness constant.

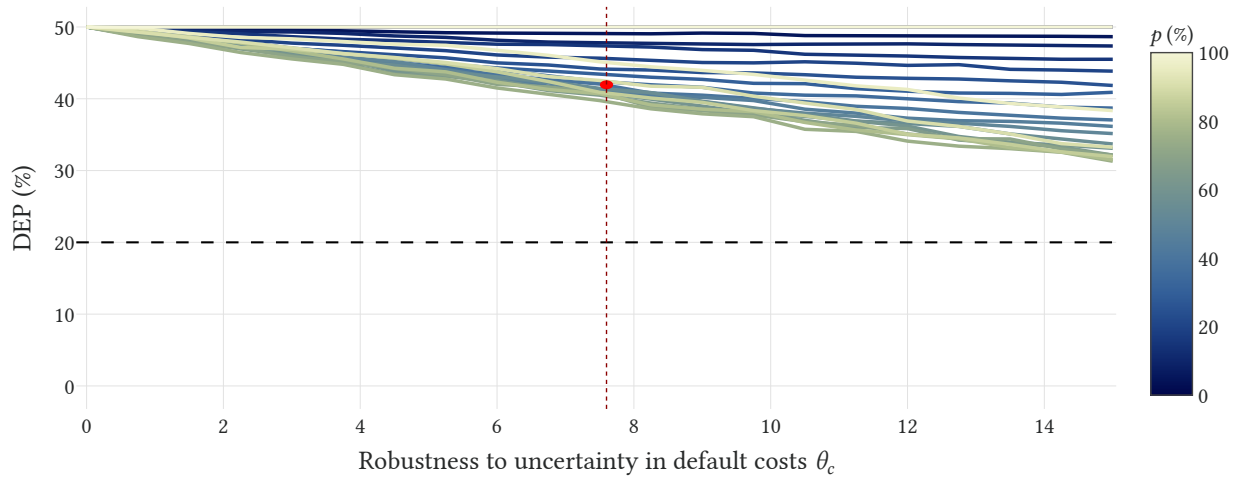


FIGURE 8: DEBT AND DEPs FOR DIFFERENT PRIOR PROBABILITIES OF A TRANSITORY COST

## 6. CONCLUDING REMARKS

This paper shows how the potential permanence of output costs following a debt restructuring can significantly amplify a country’s willingness to repay its debt and, hence, its debt capacity. Debt tolerance with fully permanent output costs can be almost one half larger than with purely transitory costs. In practice, decision makers are unlikely to have a clear picture of how permanent these costs are, similarly to researchers in sovereign debt. Aversion to this ambiguity boosts debt tolerance, by making the government treat costs as perhaps being more persistent. While these channels have received little attention in the literature, our results suggest that they may be key drivers of debt tolerance.

We find that the persistence of output costs, and the uncertainty surrounding it, has a sizable effect on debt tolerance, but perhaps not as much as one would have expected. We parametrize the model to an exclusion period of 6.5 years on average, following widely used estimates from [Cruces and Trebesch \(2013\)](#). The additional impact of making the cost permanent after the exclusion period is limited by the discounting of a relatively impatient borrower, which is also a common feature of calibrated sovereign debt models.<sup>28</sup> With a more patient borrower (or a counterfactually shorter exclusion period), permanent default costs would boost debt tolerance even more. This interaction could be a key driver of what explains differences in debt tolerance across countries (e.g. “graduation” effects).

<sup>28</sup>Such low discount factors can be interpreted as representing political-economy considerations.

The focus of this paper, like much of the literature, has been on external debt. Domestic debt restructurings can be much more disruptive, given the implications for domestic financial stability, which can potentially lead to much larger and protracted output costs relative to an external debt restructuring. Modeling domestic debt introduces complications to the model, but would be an interesting direction for future work, and where both channels proposed could have an even larger amplifying impact on debt tolerance.

## REFERENCES

- AGUIAR, M. AND M. AMADOR (2014): “Sovereign Debt,” *Handbook of International Economics*, 4, 647–87.
- (2020): “Self-Fulfilling Debt Dilution: Maturity and Multiplicity in Debt Models,” *American Economic Review*.
- AGUIAR, M., S. CHATTERJEE, H. COLE, AND Z. STANGEBYE (2016): “Quantitative Models of Sovereign Debt Crises,” *Handbook of Macroeconomics*, 2B, 1697–1755, chapter 21.
- ANDERSON, E. W., L. P. HANSEN, AND T. J. SARGENT (2003): “A Quartet of Semigroups for Model Specification, Robustness, Prices of Risk, and Model Detection,” *Journal of the European Economic Association*, 1, 68–123.
- ARELLANO, C. AND L. BARRETO (2023): “Official Sovereign Debt,” Mimeo.
- ASONUMA, T., M. CHAMON, A. ERCE, AND A. SASAHARA (2023): “Costs of Sovereign Defaults: Restructuring Strategies and the Credit-Investment Channel,” Mimeo.
- ASONUMA, T. AND C. HE (2023): ““Too Little” Sovereign Debt Restructurings,” Mimeo.
- ASONUMA, T. AND C. TREBESCH (2016): “Sovereign Debt Restructurings: Preemptive or Post-Default,” *Journal of the European Economic Association*, 14, 175–214.
- BARILLAS, F., L. HANSEN, AND T. SARGENT (2009): “Doubts or Variability?” *Journal of Economic Theory*, 144, 2388–2418.
- BORENSZTEIN, E. AND U. PANIZZA (2009): “The Costs of Sovereign Default,” *IMF Staff Papers*, 56, 683–741.
- BOZ, E. (2011): “Sovereign default, private sector creditors, and the IFIs,” *Journal of International Economics*, 83, 70–82.

- BULOW, J. AND K. ROGOFF (1989a): "Sovereign Debt: Is to Forgive to Forget?" *American Economic Review*, 79, 43–50.
- (1989b): "A Constant Recontracting Model of Sovereign Debt," *Journal of Political Economy*, 97, 155–178.
- CERRA, V. AND S. C. SAXENA (2008): "Growth Dynamics: The Myth of Economic Recovery," *American Economic Review*, 98, 439–57.
- CHATTERJEE, S., D. CORBAE, J.-V. RIOS-RULL, AND K. DEMPSEY (2018): "A Theory of Credit Scoring and the Competitive Pricing of Default Risk," 2018 Meeting Papers 550, Society for Economic Dynamics.
- CHATTERJEE, S. AND B. EYIGUNGOR (2012): "Maturity, Indebtedness, and Default Risk," *American Economic Review*, 102, 2674–99.
- COLE, H. AND P. KEHOE (1998): "Models of Sovereign Debt: Partial versus General Reputations," *International Economic Review*, 39, 55–70.
- CRUCES, J. J. AND C. TREBESCH (2013): "Sovereign Defaults: The Price of Haircuts," *American Economic Journal: Macroeconomics*, 5, 85–117.
- DE PAOLI, B., G. HOGGARTH, AND V. SAPORTA (2009): "Output costs of sovereign crises: some empirical estimates," Bank of England working papers 362, Bank of England.
- D'ERASMO, P., E. MENDOZA, AND J. ZHANG (2016): "Quantitative Models of Sovereign Debt Crises," *Handbook of Macroeconomics*, 2B, 2493–2597, chapter 32.
- DVORKIN, M., J. M. SÁNCHEZ, H. SAPRIZA, AND E. YURDAGUL (2021): "Sovereign Debt Restructurings," *American Economic Journal: Macroeconomics*, 13, 26–77.
- EATON, J. AND M. GERSOVITZ (1981): "Debt with Potential Repudiation: Theoretical and Empirical Analysis," *The Review of Economic Studies*, 48, 289–309.
- FARAH-YACOUB, J. P., C. M. H. GRAF VON LUCKNER, R. RAMALHO, AND C. M. REINHART (2022): "The Social Costs of Sovereign Default," Policy Research Working Paper Series 10157, The World Bank.
- FOURAKIS, S. (2023): "Sovereign Default and Government Reputation," Mimeo.

- HANSEN, L. P. AND T. J. SARGENT (2001): “Robust Control and Model Uncertainty,” *American Economic Review*, 91, 60–66.
- (2015): “Four types of ignorance,” *Journal of Monetary Economics*, 69, 97–113.
- HATCHONDO, J. C. AND L. MARTINEZ (2009): “Long-duration bonds and sovereign defaults,” *Journal of International Economics*, 79, 117–125.
- HATCHONDO, J. C., L. MARTINEZ, AND Y. K. ONDER (2017): “Non-defaultable debt and sovereign risk,” *Journal of International Economics*, 105, 217–229.
- IMF (2013): “Sovereign Debt Restructuring - Recent Developments and Implications for the Fund’s Legal and Policy Framework,” available at: [https://www.imf.org/-/media/Websites/IMF/imported-full-text-pdf/external/np/pp/eng/2013/\\_042613.ashx](https://www.imf.org/-/media/Websites/IMF/imported-full-text-pdf/external/np/pp/eng/2013/_042613.ashx), IMF Policy Papers.
- (2020): “The International Architecture for Resolving Sovereign Debt Involving Private-Sector Creditors—Recent Developments, Challenges, And Reform Options,” available at: <https://www.imf.org/-/media/Files/Publications/PP/2020/English/PPEA2020043.ashx>, IMF Policy Paper No. 2020/043.
- (2021): “Issues in Restructuring of Sovereign Domestic Debt,” available at: <https://www.imf.org/-/media/Files/Publications/PP/2021/English/PPEA2021071.ashx>, IMF Policy Paper No. 2021/071.
- KARANTOUNIAS, A. G. (2023): “Doubts about the model and optimal policy,” *Journal of Economic Theory*, 210, 105643.
- LELAND, H. E. (1998): “Agency Costs, Risk Management, and Capital Structure,” *Journal of Finance*, 53, 1213–1243.
- LEVY YEYATI, E. AND U. PANIZZA (2011): “The elusive costs of sovereign defaults,” *Journal of Development Economics*, 94, 95–105.
- MARTINEZ, L., F. ROCH, F. ROLDÁN, AND J. ZETTELMEYER (2023): “Sovereign Debt,” in *The Research Handbook of Financial Markets*, ed. by R. S. Gürkaynak and J. H. Wright, Edward Elgar Publishing, chap. 17, 378–405.
- MEYER, J., C. M. REINHART, AND C. TREBESCH (2022): “Sovereign Bonds since Waterloo,” *The Quarterly Journal of Economics*, 137.

- MIHALACHE, G. (2020): “Sovereign default resolution through maturity extension,” *Journal of International Economics*, 125C, 103326.
- PÉREZ, D. (2018): “Sovereign Debt, Domestic Banks and the Provision of Public Liquidity,” mimeo, NYU.
- POUZO, D. AND I. PRESNO (2016): “Sovereign Default Risk and Uncertainty Premia,” *American Economic Journal: Macroeconomics*, 8, 230–66.
- REINHART, C., K. ROGOFF, AND M. A. SAVASTANO (2003): “Debt Intolerance,” *Brookings Papers on Economic Activity*, 34, 1–74.
- REINHART, C. M. AND K. S. ROGOFF (2011): “The Forgotten History of Domestic Debt,” *Economic Journal*, 121, 319–350.
- ROCH, F. AND F. ROLDÁN (2023): “Uncertainty Premia, Sovereign Default Risk, and State-Contingent Debt,” *Journal of Political Economy Macroeconomics*, 1, 334–370.
- ROGOFF, K. (2022): “Issues in the theory of sovereign debt and post-covid workouts,” *Journal of Policy Modeling*, 44, 804–811.
- ROLDÁN, F. AND C. SOSA-PADILLA (2023): “Central Bank Swap Lines as Bilateral Sovereign Debt,” Mimeo.
- STRZALECKI, T. (2011): “Axiomatic Foundations of Multiplier Preferences,” *Econometrica*, 79, 47–73.

## A. OUTPUT AROUND RESTRUCTURING EVENTS

Figure A.1 presents the same output deviations from Figure 1 but in calendar time, and also including those restructuring events occurring before 1990, which are associated with higher volatility in outcomes.

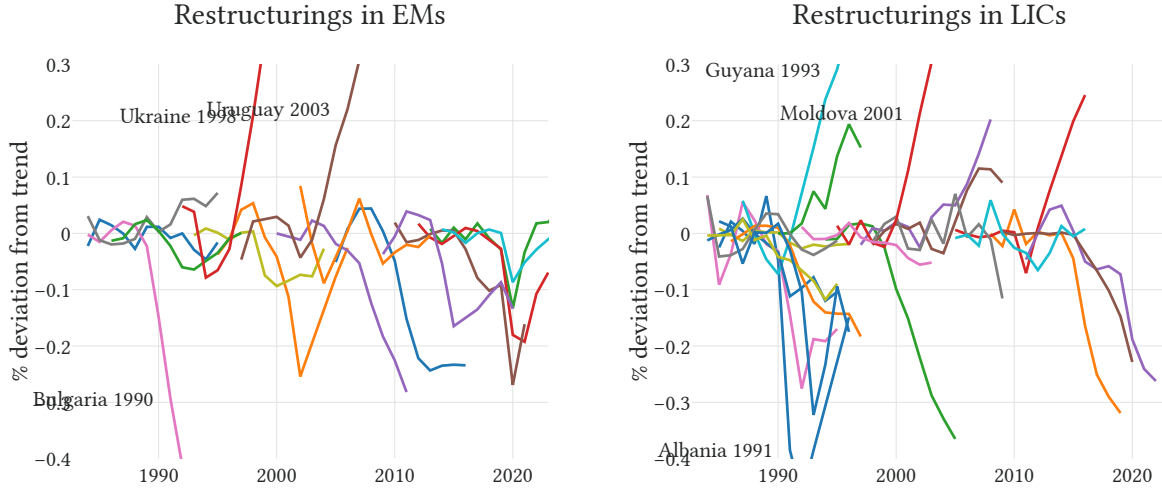


FIGURE A.1: GROWTH OUTCOMES AROUND DEBT RESTRUCTURINGS

*Note:* Outlier trajectories censored and marked

The events in our sample for EMs (or Market-Access Countries) are Argentina 2001, Argentina 2019, Barbados 2018, Belize 2006, Belize 2016, Ecuador 1999, Ecuador 2020, Greece 2011, Seychelles 2008, South Africa 1992, Ukraine 1998, Ukraine 2015, and Uruguay 2003. Those classified as LICs are Albania 1991, Chad 2014, Côte d’Ivoire 2000, Côte d’Ivoire 2011, Dominica 2003, Dominican Republic 2004, Grenada 2004, Guinea 1991, Guyana 1993, Kenya 1992, Mauritania 1992, Moldova 2001, Mongolia 2017, Mozambique 2015, Pakistan 1998, St. Kitts and Nevis 2011, and Togo 1991.

## B. NORMALIZATION DETAILS

We normalize all variables by  $\Gamma_t$ , denote normalized values with lowercase, and notice that  $y_t = \exp(z_t)$  and

$$u(C) = \frac{C^{1-\gamma}}{1-\gamma} = \Gamma^{1-\gamma} \frac{(C/\Gamma)^{1-\gamma}}{1-\gamma} = \Gamma^{1-\gamma} u(c)$$

so in a typical Bellman equation we can guess and verify (denoting  $x = B'/\Gamma$ ) forms like  $V\Gamma^{\gamma-1} = v$

$$\begin{aligned} V(B, z, \Gamma) &= \max_{B'} u(C) + \beta \mathbb{E} [V(B', z', \Gamma')] \\ V(B, z, \Gamma) &= \max_{B'} \Gamma^{1-\gamma} u(c) + \beta \mathbb{E} [V(B', z', \Gamma')] \\ \Gamma^{\gamma-1} V(B, z, \Gamma) &= \max_{B'} u(c) + \beta \mathbb{E} \left[ (\Gamma')^{\gamma-1} (\Gamma/\Gamma')^{\gamma-1} V(B', z', \Gamma') \right] \\ v(b, z, g) &= \max_x u(c) + \beta \mathbb{E} \left[ (g')^{1-\gamma} v(b'(x, g'), z', g') \right] \\ v(b, z, g) &= \max_x u(c) + \beta \mathbb{E} \left[ (g')^{1-\gamma} v(b'(x, g'), z', g') \right] \end{aligned}$$

while for the budget constraint we have

$$\begin{aligned} C + \kappa B &= Y + q(B' - (1 - \rho)B) \\ c + \kappa b &= y + q(B'/\Gamma - (1 - \rho)b) \\ c + \kappa b &= y + q(b'(\Gamma'/\Gamma) - (1 - \rho)b) \\ c + \kappa b &= y + q(b'g' - (1 - \rho)b) \end{aligned}$$

This budget constraint makes it clear that  $x = b'g'$  (simply substituting  $x = B'/\Gamma$  in line 2) or  $b'(x, g') = x/g'$ .

### C. ROBUSTNESS

For a technical exposition of robustness, see [Hansen and Sargent \(2001\)](#). Less formally, consider a decision maker facing a state (vector)  $x$  with a (trusted) law of motion  $\mu(x' | x, c)$  and a Bellman equation of the form

$$v(x) = \max_c u(c) + \beta \int v(x') \mu(x' | x, c) dx'$$

To include doubts about the law of motion  $\mu$  and guard against misspecification, the decision maker seeks an action that would perform reasonably well if the true law of motion was instead *any*  $\mu'$ . The alternative is left unidentified but could be any probability distribution that is difficult to distinguish statistically from  $\mu$ . To obtain an action that performs well under all possible  $\mu'$ , the decision maker enlists the help of an auxiliary evil agent who chooses a probability distortion  $m$  that minimizes the utility given the action  $c$  chosen. By behaving in this pessimistic way (each action is evaluated according to the distortion that yields the lowest utility), the decision maker obtains lower bounds for each action.

Finally, in order to only allow distortions that are difficult to identify from data, the evil agent faces a cost for the relative entropy of the original model  $\mu$  and the distorted one, the so-called



worst-case model  $m\mu$ . The inverse marginal cost of relative entropy,  $\theta$ , determines how robust the resulting decision will be. The decision maker takes into account the reaction of the evil agent and solves

$$v(x) = \max_c \min_{m \geq 0} u(c) + \beta \int v(x') m(x') \mu(x' | x, c) dx' + \frac{1}{\theta} \text{ent}(m\mu, \mu)$$

subject to  $\int m(x') \mu(x' | x, c) dx' = 1$

The solution to the minimization problem yields a worst-case model  $\hat{m}(x') \propto \exp(-\theta v(x'))$ . As [Hansen and Sargent \(2001\)](#) show, plugging in the worst-case model yields the robust Bellman equation

$$v(x) = \max_c u(c) + \frac{1}{-\theta} \beta \log \left( \int \exp(-\theta v(x')) \mu(x' | x, c) dx' \right)$$

#### D. SOLUTION ALGORITHM

We focus on Markov Perfect equilibria which we find as the limit of a finite-horizon version of the model to handle possible multiplicity.

We start at time  $T$  from a policy of always repaying the debt before the world ends, meaning that  $v(b, z) = v_R(b, z)$  is given by the utility of consuming the endowment net of coupon payments  $y(z) - \kappa b$ . Because the world ends, at  $T$  the price of debt is  $q(x, z) = 0$  in all states.

We iterate backward in time. At each period, we first use the default probabilities  $\mathcal{P}$  and issuance  $x'$  policies, along with the current guess of the price  $q$ , to compute the new debt price from (13). Then with new debt prices in hand and the current guess of the value function  $v$  we move to updating  $v_R$  and  $x'$  from (14). Using the guesses for  $v$ ,  $v_D^T$  and  $v_D^P$ , we can also update the values of default  $v_D^T$  and  $v_D^P$  from (16) and subsequently  $v_D$  from (15). Finally, with updated values for default and repayment, we update the beginning-of-period value function  $v$  as well as the default probability  $\mathcal{P}$  from (9).

The preference shocks, as represented by the closed forms (9), produce enough smoothing in the value function to allow this algorithm to converge, while avoiding the problems highlighted by [Chatterjee and Eyigungor \(2012\)](#). Our formulation with preference shocks produces a similar smoothing as the two-tiered income process introduced by [Chatterjee and Eyigungor \(2012\)](#) and commonly used in discrete state space solutions of Eaton-Gersovitz models, but the closed-form solutions for  $\mathcal{P}$  and  $v$  avoid the computational complication introduced by the inclusion of the extra state required by a two-tiered income process.