

The Optimal-Contracting Approach in Macroeconomics

NOTES FROM A CLASS TAUGHT BY ALESSANDRO DOVIS AND TOM SARGENT AT NYU IN THE FALL OF 2017

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1 Calvo 1978 / Chang 1998

Let $m_t = q_t M_t$ denote real money balances, for $q_t = 1/p_t$. The government collects taxes x_t to inject or withdraw money from the system: $q_t(M_t - M_{t-1}) = -x_t$.

The household's problem is

$$\begin{aligned} \max_{c_t, M_t} \sum_{t=0}^{\infty} \beta^t \left(u(c_t) + v(\underbrace{q_t M_t}_{= m_t}) \right) \\ \text{subject to } q_t M_t + c_t = y_t + q_t M_{t-1} - x_t \\ q_t M_t \leq \bar{m} \end{aligned} \tag{1}$$

Assuming that $y_t = f(x_t)$ to summarize distortions from taxation, the household's first-order conditions are

$$\begin{aligned} u'(c_t) &= \lambda_t \\ q_t \left[u'(c_t) - v'(m_t) \right] &\leq \beta u'(c_{t+1}) q_{t+1} \quad = \text{if } m_t < \bar{m} \end{aligned} \tag{2}$$

Replacing the government's budget constraint yields $c_t = y_t = f(x_t)$, so the first-order condition for money holdings becomes (after also multiplying by M_t on both sides)

$$m_t \left[u'(c_t) - v'(m_t) \right] \leq \beta \underbrace{u'(f(x_{t+1})) m_{t+1} h_{t+1}}_{= \theta_{t+1}} \quad = \text{if } m_t < \bar{m} \tag{4}$$

where $h_t = M_{t-1}/M_t$.

θ_{t+1} summarizes all relevant aspects of (expected) future outcomes that the household needs to make decisions at t .

We can rewrite (4) as

$$\theta_t \leq u'(f(x_t)) + v'(m_t)m_t + \beta\theta_{t+1} \quad = \text{if } m_t < \bar{m} \quad (5)$$

The government's (or planner's) problem is to then choose a stochastic process for x_t subject to the household's equilibrium response. In other words, the government picks an $\{x_t\}_t$, solves for θ , obtains the competitive equilibrium allocation, and can use it to evaluate the welfare of having chosen $\{x_t\}_t$.

Households think they are choosing m_t , they are being manipulated by the Ramsey planner.

1.1 Recursive planner's problem

It turns out that it is possible to reexpress the planner's problem as solving the following two subproblems and associated Bellman equations

Subproblem 1 Let θ be an artificial state variable which for unknown reasons the current decision maker perceives as a constraint; and such that the current decision maker has a technology to send θ' to the following decision maker

$$\begin{aligned} J(\theta) &= \max_{x, m, \theta'} u(f(x)) + v(m) + \beta J(\theta') \\ \text{subject to } \theta &= u'(f(x))x + v'(m)m + \beta\theta' \\ \theta' &\in \Omega \end{aligned} \quad (6)$$

for some suitable set Ω

[What is the appropriate set Ω ?]

The solution to (6) yields optimal decision rules $x(\theta)$, $m(\theta)$ and $\theta' = g(\theta)$.

The telltale sign of time inconsistency is that the decision maker chooses to change the message: $g(\theta) \neq \theta$.

Subproblem 2 Choosing the initial θ

$$J_0 = \max_{\theta \in \Omega} J(\theta) \quad (7)$$

1.2 Finding the state is an art

Let $w_t = u(f(x_t)) + v(m_t) + \beta w_{t+1}$ and notice that, along the path of the Ramsey plan,

$$w_t = u(f(x(\theta_t))) + v(m(\theta_t)) + \beta w_{t+1}.$$

We can then think of the pair (w_t, θ_t) as the system's state, where $\theta_t = g(\theta_{t-1})$ summarizes the past and w_t summarizes the future.

[Maybe include the incentive constraint.]

2 Chari-Kehoe 1990 / Phelan-Stacchetti 2001

Imagine a situation in which some action (like capital taxation) is non-distortionary or otherwise optimal ex-post but harmful ex-ante, so it would be desirable to promise not to use it.

Can use the reaction function of the small agents to eliminate the **instruments** and choose **allocations** directly.

Phelan and Stacchetti apply Chari and Kehoe's sustainable plans to a neoclassical growth model. This sets up a dynamic game in which the capital stock k is a state variable.

Planner's problem

$$\begin{aligned} \max \quad & \sum_{t=0}^{\infty} \beta^t (u(c_t, n_t) + v(g_t)) \\ \text{subject to} \quad & c_t + g_t + k_{t+1} \leq F(k_t, n_t) \\ & g_t \leq \tau_t^\ell w_t n_t + \tau_t^k R_t k_t \end{aligned} \tag{8}$$

Imagine that the planner chooses (τ_t^ℓ, τ_t^k) each period, while the household chooses labor supply n and savings θ , which determine consumption via the budget constraint

$$c = (1 - \theta) \left[(1 - \tau_t^k) R_t k_t + (1 - \tau_t^\ell) w_t n_t \right] \tag{9}$$

Now take the firm's first-order conditions to eliminate prices

$$R_t = F_K(k_t, n_t), \quad w_t = F_N(k_t, n_t) \tag{10}$$

To define sustainable plans in this economy, define the *public histories* as

$$h^t = (h^{t-1}, \pi_t), \quad \pi_t = (\tau_t^k, \tau_t^\ell)$$

and strategies for the government and the household, respectively,

$$\begin{aligned} \sigma_g(h^{t-1}) &= \pi_t \\ \sigma_{hh}(h^{t-1}, \pi_t) &= (c_t, \theta_t, n_t, k_{t+1}) \end{aligned}$$

The household strategy can depend on the government's current action. This makes the government a Stackelberg leader.

Let the government/planner's value be denoted by $v(\sigma|h^{t-1})$ after history h^{t-1} .

Definition 2.1. A *sustainable equilibrium* is a strategy σ such that

1. For all $\tilde{\sigma}_g, h^{t-1}$, $v(\sigma_g, \sigma_{hh}|h^{t-1}) \geq v(\tilde{\sigma}_g, \sigma_{hh}|h^{t-1})$
2. The household's strategy σ_{hh} is a competitive equilibrium (i.e., satisfies first-order conditions, budget constraints, etc) given σ_g .

Now, let $m_{t+1} = u_c(c_{t+1}, n_{t+1})(1 - \tau_{t+1}^k)R_{t+1}$ be the marginal utility (realized at $t + 1$) of a marginal unit of savings at time t .

Imagine an auxiliary problem in which the household maximizes with preferences given by $u(c_t, n_t) + \beta m_{t+1}k_{t+1}$ and is subject to the budget constraint. Let $CE(k, \pi, m)$ denote the set of all possible private-sector equilibrium actions (consistent with first-order conditions, and budget constraints), at state k when the government's action is π and expectations are m . It is easy to see that

$$\left(\underbrace{c_t, n_t, k_{t+1}}_{=x_t}, \underbrace{(R_t, w_t)}_{=p_t} \right) \in CE(k_t, \pi_t, m_{t+1})$$

if

$$U_{Ct} = \beta m_{t+1} \quad \text{and} \quad U_{Nt} = U_{Ct}(1 - \tau_t^\ell)F_N(k_t, n_t)$$

Lemma 2.1. $(x, p, \pi) \in CE(k_0)$ iff $(x_t, p_t) \in CE(k_t, \pi_t, m_{t+1})$ for all t , where $m_{t+1} = U_{Ct+1}(1 - \tau_{t+1}^k)R_{t+1}$.

APS Defining the operators

$$\begin{aligned} V(K_0) &= \{v(\sigma, K_0) : \sigma \in \text{SUST}(k_0)\} \\ W(K_0) &= \{(v, m) : \exists \sigma \in \text{SUST}(k_0) \text{ s.t. } v = v(\sigma, K_0), \\ &\quad m = U_{C0}(1 - \tau_0^k(\sigma))F_{K,0}(\sigma, k_0)\} \end{aligned}$$

all of the theorems from Abreu-Pearce-Stacchetti (factorization, self-generation) generalize to this setting with state variables.

3 Alvarez-Jermann 2000

This is an economy in which default is possible but never happens in equilibrium, as a result of perfect information and liability design (since default is costly, it is avoided when possible).

Here the key is dynamic programming squared: using the other player's Bellman equation as a state in the planner's problem.

$$\begin{aligned}
 J(v, s_0) &= \max_{c(s), c^*(s), v'(s)} \sum_s \pi(s|s_0) [u(c(s)) + \beta J(v'(s), s)] \\
 \text{subject to } &\sum_s \pi(s|s_0) [u(c^*(s)) + \beta v'(s)] = v \\
 &c(s) + c^*(s) = y(s) + y^*(s) \\
 &u(c(s)) + \beta J(v'(s), s) \geq u(y(s)) + \beta v_{\text{aut}}(s) \\
 &u(c^*(s)) + \beta v'(s) \geq u(y^*(s)) + \beta v_{\text{aut}}^*(s) \\
 &v'(s) \in [v_{\text{aut}}^*(s), v_{\text{max}}(s)]
 \end{aligned} \tag{11}$$

where $v_{\text{max}}(s)$ is implicitly defined by $J(v_{\text{max}}(s), s) = v_{\text{aut}}(s)$.

The first constraint is *promise-keeping* and states that the value v must be delivered to the agent. The second constraint is a feasibility constraint. The third and fourth are *participation* constraints, ensuring that both agents prefer to remain in the contract rather than retaining their current endowment and remaining in autarky from then on. The last constraint is an ad-hoc way of avoiding Ponzi schemes sustained by unbounded payoffs.

While the operator in (11) is not a contraction, it is monotone. This means that it is possible to start from a very high guess for J and converge to the fixed point from above.