On the Optimal Speed of Sovereign Deleveraging with Precautionary Savings*

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Abstract

We study the interaction between sovereign risk and aggregate demand driven by the endogenous response of savers to sovereign risk. We obtain two main results. First, this new sovereign risk / aggregate demand channel creates a tradeoff between the recessionary impact of fiscal consolidation and the risk of a future sovereign debt crisis. Risk aversion has a large impact on output losses and on welfare when sovereign debt is risky. Second, we find that savers and borrowers disagree about the optimal path of sovereign deleveraging. The sovereign risk channel can therefore explain some of the rise in political disagreement about fiscal policy. Using a version of the model calibrated to the Eurozone crisis, we find that sovereign risk justifies starting to deleverage about two years after the beginning of the recession. We also find that, after 2012, the channel was weakened so that active deleveraging in a recession is no longer justified.

JEL: E2, G2, N2

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How fast should governments reduce their leverage? The question has been at the center of much policy debate in the aftermath of the Great Recession and of the Eurozone crisis. One side of the tradeoff is rather straightforward. In a non-Ricardian world, fiscal consolidation can depress aggregate demand and decrease employment. Whether this is good or bad depends on the state of the economy. If the economy is already depressed, fiscal consolidation is pro-cyclical and it can have large negative welfare consequences.

The other side of the trade-off is more complex to analyze. We take the view that the goal of fiscal consolidation is to lower future sovereign default risk. The key issue is then to understand how sovereign risk affects the economy. If domestic agents hold government debt, then the presence of default risk can induce precautionary behavior and become another source of insufficient aggregate demand. In benchmark models of external debt the cost of default is a temporary exclusion from international financial markets. But exclusion does not last long in practice, and official lenders typically step in when private lenders pull out. The literature then usually assumes that sovereign default creates a large exogenous loss in output (Eaton and Gersovitz, 1982; Arellano, 2008).

A more recent literature, motivated in part by the Eurozone crisis, has emphasized the impact of sovereign risk on the funding costs of financial intermediaries. Sovereign credit risk can directly hurt levered financial intermediaries, either because they hold government bonds (Gennaioli et al., 2014; Bocola, 2016; Perez, 2016), or because the state insures some of their liabilities (e.g., deposit insurance). Gourinchas et al. (2016) emphasize a different channel, austerity-induced non-performing loans. They assume a fiscal rule where the government cuts spending and/or raises taxes when sovereign risk increases. Fiscal austerity lowers output and increases credit risk in the private sector, which hurts the banks.

All these channels are relevant, but all of them are also limited. The financial channel is plausible in the short run, but it only operates if one assumes that intermediaries cannot raise capital. If banks can raise equity then what matters is the pricing kernel of their shareholders. Moreover, most of the public debt is not held by banks or levered institutions, but rather by pension funds and in separate accounts of insurance companies. It is therefore important to consider models where domestic sovereign exposure is not concentrated in levered financial institutions with exogenous capital, but rather borne, directly or indirectly, by domestic savers. Models that assume a fiscal rule, such as Martin and Philippon (2017) or Gourinchas et al. (2016), are not satisfactory either because they simply assume that governments impose austerity in response to sovereign risk. In this paper, by contrast, we study the optimal speed of sovereign deleveraging.

Assuming that sovereign debt is held by risk-averse domestic savers turns out to deliver rich dynamics and new insights. First, it clarifies the role of non-Ricardian features. We usually think of Ricardian equivalence as saying that the timing of taxes does not matter, but in fact it also says that sovereign risk does not matter. A direct corollary of Ricardian equivalence is that sovereign defaults on domestic debt are irrelevant, both ex-ante and ex-post. In a Ricardian model, if the government imposes a haircut of 30% on its debt, nothing happens, because this is exactly compensated by a commensurate decrease in the net present value of taxes.

We consider a model where some agents are constrained in their ability to borrow, which breaks Ricardian equivalence and activates both sides of the tradeoff. On the one hand, an increase in taxes used to repay the debt of the government has a negative impact on the disposable income of constrained agents, and thus on aggregate demand for goods and services.

On the other hand, the larger is the debt of the government, the higher is the probability of a default. Government default represents a net loss for holders of government debt. In our model the holders of government debt are domestic savers. The risk of sovereign default increases their precautionary motive for savings and hurts aggregate demand. An important point here is that this effect is linked to the size of the expected potential financial loss of savers, even if default does not create exogenous deadweight losses.

We therefore obtain a new tradeoff between the contractionary effects of fiscal consolidation and those from the risk of a sovereign debt crisis. When the risk of default is very responsive to the level of debt, our model predicts that austerity can be expansionary. An increase in taxes can lead to a decrease in precautionary savings that is strong enough to offset the direct effect on the disposable income of constrained agents.

To illustrate, we study a sequence of shocks that captures the timing of events during the Great Recession and the Eurozone crisis. The economy starts in steady state. The first shock marks the start of private deleveraging which forces constrained agents to pay back some of their debts. Aggregate demand drops, as in Eggertsson and Krugman (2012). We consider an economy where wages are sticky and the nominal interest rate does not adjust, either because it is set outside the country (eurozone) or because of the zero lower bound. Private deleveraging then creates a recession.

The second shock that hits the economy is a sovereign risk shock, modeled as an increase in the perceived risk of government default. At this point we ask what is the optimal path for sovereign deleveraging. The government can start immediately, or it can wait until private deleveraging is over. We estimate output losses for each strategy and the welfare of savers and borrowers. Expansionary austerity does not arise in our calibrated model but a clear feature of the simulations is that borrowers and savers disagree about the path of deleveraging. To understand why, notice that sovereign risk is the sum of two shocks. On the one hand, sovereign default redistributes wealth from savers, who are both taxpayers and holders of debt, to borrowers, who are only taxpayers. This is the source of disagreement. On the other hand, expected default risk increases precautionary savings and lowers demand and output, which is a deadweight loss for all the agents. If this effect is very strong, the two types are more likely to agree. In general, borrowers prefer delayed deleveraging, while savers prefer early deleveraging, even though they understand that this will reduce their current labor earnings. Our model can therefore shed light on the political tensions that have appeared in most countries regarding fiscal policy after the Great Recession. A relevant point here is that the model predicts that disagreement is maximum not during the peak of the financial crisis, but rather later when spreads have gone down.

Before presenting the full dynamic model, we use a stylized two-period model to build intuition. There we can easily characterize the amount of sovereign deleveraging that maximizes the welfare of borrowers and savers. Announcing a reduction of debt has an immediate effect on the probability of default, decreases precautionary savings and boosts aggregate demand. When this effect is strong enough, we obtain expansionary austerity. Expansionary austerity is a theoretical possibility of our model but we find that it does not happen in the calibrated multi-period model. In other words, in the calibrated model there is a trade-off between austerity and sovereign risk. We then use the model to study the optimal timing of fiscal consolidation.

Discussion of the Literature The literature on sovereign debt usually assumes that sovereign bonds are priced by deep-pocket investors, often risk-neutral and interpreted as international lenders. In the benchmark models of Aguiar and Gopinath (2006) and Arellano (2008), for instance, the government trades one period discount bonds with risk neutral competitive foreign creditors. As a result the price of the bond is $q_t = \frac{1-\pi_t \hbar}{1+r}$ where π is the probability of default and $\hbar$ is the haircut in case of default. The assumption that matters is not that investors are risk neutral, since we can always reinterpret the model as being written under the risk neutral measure of foreign lenders or, equivalently, assume that their pricing kernel is correlated with the country's risk.[1]

By contrast, our key mechanism is that sovereign debt is held (in large parts) by *domestic* agents who are risk averse. We consider a model with Epstein and Zin (1989) preferences to distinguish risk aversion from inter-temporal substitution. In addition, precautionary savings interact with the non-Ricardian features, so that agents are not only averse to deadweight losses but also to haircuts, which is clearly an important empirical feature. In that sense our model resembles the models where sovereign risk hurts directly the balance sheet of levered financial intermediaries (Gennaioli et al., 2014; Bocola, 2016; Perez, 2016). Bocola (2016) models the direct exposure of banks. He decomposes the impact in two channels. First, asset losses can create a binding constraint on banks, leading to a decline in credit supply. But there is also a precautionary channel: even if the funding constraint of banks is not

[1] Arellano (2008) extends her basic framework to risk averse lenders and chooses the parameters of their pricing kernel to match the average spread.

currently binding, it might bind in the future, and banks can decide to reduce their lending as a precautionary measure. The main result in Bocola (2016) is that the precautionary channel can be significant (up to 40% of the entire effect).

The predictions of models based on levered intermediaries' exposures, however, are very sensitive to the details of financial contracts available to intermediaries. Amplification only happens when intermediaries issue non-contingent debt and cannot be recapitalized. In the short run the assumption of constant bank capital is realistic, but less so as time passes. Our model can thus be thought of as a medium run model of sovereign risk.

Another critical difference with the existing literature is that our model combines sovereign risk with nominal rigidities. Sticky prices generate a role for policy to manage aggregate demand and the level of output. Models of aggregate demand (Campbell and Mankiw, 1989; Curdia and Woodford, 2009; Gali, 2008) analyze nominal rigidities together with non-Ricardian features, but think about shorter-term issues than sovereign debt management. On the other hand, many studies of sovereign risk include non-Ricardian features as a by-product of distributional concerns (see Woodford, 1990; Aiyagari and McGrattan, 1998, and the literature building on them) but abstract from aggregate demand considerations.

To keep the model tractable, we assume an exogenous mapping from debt levels to default risk, while much of the literature focuses on the incentives to repay, as summarized in Aguiar and Amador (2014). In particular, Aguiar et al. (2016) consider a standard small open economy model and study the role of short versus long term debt in the adjustment process. Their main result is that, because of endogenous price reactions, it is never optimal for the government to actively trade in its own long term debt. The government should retire old bonds as they mature and should perform all the adjustment by changing the path of issuance of short term debt. Our setup is different because debt is held domestically and the endogenous reaction of savers has an impact on aggregate demand. On the other hand, our model has nothing to say about the maturity of debt because we assume that the risk of default is an exogenous function of the level of debt.

Our findings on the effects of risk aversion differ sharply from Tallarini (2000)'s macro-finance separation result. He finds that, in a real business cycle model with Epstein-Zin preferences, macroeconomic dynamics depend on the elasticity of inter-temporal substitution (EIS), but not on the coefficient of risk aversion. Asset prices, on the other hand, depend on risk aversion. As a result, there is a separation between macroeconomics and finance. We obtain a very different result in our model. Holding the EIS constant, we find that, beyond some level of risk aversion, the price of government debt is not very sensitive to risk aversion. Instead, more risk-averse savers cut their consumption more in response to credit risk, so that the actual default outcome becomes relatively less severe for them. This cancels out the direct impact of risk aversion on sovereign spreads. In our model, an increase in risk aversion has a large impact on macroeconomic dynamics, and a somewhat weaker impact on asset prices.

The literature has also analyzed the feedback from private credit risk to sovereign risk. There are also two main channels: a macroeconomic channel, and a financial guarantee channel. The macro channel is straightforward: an increase in private funding costs decreases investment and consumption by borrowers, which can lead to a recession and lower tax revenues, more transfer payments on automatic programs (e.g. unemployment insurance) and perhaps discretionary fiscal stimulus, all of which can increase sovereign debt (Martin and Philippon, 2017; Gourinchas et al., 2016). The guarantee channel applies mostly to explicit and implicit guarantees on financial intermediaries, ranging from deposit insurance to outright bailouts (Acharya et al., 2015).

The distribution of wealth and income also shapes the decisions surrounding sovereign debt policy, as has been emphasized since Woodford (1990) and Aiyagari and McGrattan (1998). D'Erasmo and Mendoza (2016) build a heterogeneous-agents model of sovereign default and find that levels of debt like those of present day Spain suggest a government with a bias towards favoring its creditors. Ferriere (2016) and Ferriere and Navarro (2017) argue for a positive link between progressive taxation on the one hand and incentives to repay sovereign debt and fiscal multipliers on the other. Guembel and Sussman (2009) and Andreasen et al. (2011) study political economy considerations in sovereign debt policy, while Dovis et al. (2016) find that, in an overlapping-generations economy, the tension between the ex-ante desire to promote savings and the ex-post temptation to redistribute by taxing capital can lead to 'populist cycles' of austerity and external debt-financed expansions.

Our tradeoff between causing a recession and risking a debt crisis hinges crucially on the size of the fiscal multiplier, which is an endogenous object. Recent research by Huidrom et al. (2016) and Huidrom et al. (2016) points to the level of government debt and the 'fiscal space' as central determinants of the multiplier, aside from the cycle. They find low or even negative multipliers when government debt is high. Our results suggest that precautionary behavior could be behind such low multipliers when sovereign default becomes a clear possibility. Also closely related is the work of Romei (2015), who looks at a similar problem of a government deciding how fast to pay down a given stock of debt. However, she is mostly interested in the distributional aspects of this deleveraging and not in the decision of how long to remain in a crisis-prone region (Cole and Kehoe, 2000), which is the focus here. This is also related to the recent work of Escolano and Gaspar (2016)

The remainder of the paper is organized as follows. Section 1 describes the macroeconomic setup of the model. Section 2 develops a simple 2-period model with CARA preferences to build intuition for our results. Section 3 presents the full model with Epstein-Zin preferences. Section 4 concludes.

1 General Setup

We consider a closed economy that nevertheless faces an exogenous real risk-free interest rate, which we interpret either as a binding zero lower bound or as the closed economy *limit* of a small open economy running a hard peg on its nominal exchange rate. This section introduces the basic features of our model.

1.1 Government

The government spends $G_{\$,t}$ on goods and services, levies lump-sum taxes T_t, and issues long term bonds. We model bonds with geometrically decaying face value as in Leland (1998).

One unit of face value debt issued at time t pays a coupon $(1-\rho)^s\kappa$ in period $t+s+1$ as long as the government does not default. Let $B^g_{\$,t}$ be the face value in units of debt outstanding at the end of time t. Because debt decays at rate ρ, the amount of debt brought from the past is $(1-\rho)B^g_{\$,t-1}$. The net issuance is therefore $B^g_{\$,t}-(1-\rho)B^g_{\$,t-1}$. The appealing feature of Leland (1998) is that all debt trades at the same unit price, irrespective of when it was issued. Let q_t be the price of one unit of government debt. The nominal budget constraint of the government, conditional on not defaulting, is

$$q_t\left(B^g_{\$,t}-(1-\rho)B^g_{\$,t-1}\right)=\kappa B^g_{\$,t-1}+P_t\left(G_{\$,t}-T_{\$,t}\right),$$

where P_t is the price index of the economy. It will be convenient to work with real variables, so we define real government debt $B^g_t\equiv\frac{B^g_{\$,t}}{P_t}$. We can then re-write the budget constraint (conditional on not defaulting) as

$$q_t\left(B^g_t-(1-\rho)\frac{B^g_{t-1}}{\Pi_t}\right)=\kappa\frac{B^g_{t-1}}{\Pi_t}+G_t-T_t, \tag{1}$$

where $\Pi_t\equiv\frac{P_t}{P_{t-1}}$ is the inflation rate from $t-1$ to t. This formula makes it clear that unexpected inflation, if any, at time t would lower the real debt burden. We use this convention for all other nominal assets.

The return from holding performing debt between t and $t+1$ is

$$\tilde{R}^{(g,0)}_{t+1}=\frac{\kappa+(1-\rho)\,q_{t+1}}{q_t}$$

Let r be the (constant) global risk free rate. The price $q^\star$ of risk-free debt must satisfy $\tilde{R}^{(g,0)}=1+r$ so

$$q^\star=\frac{\kappa}{r+\rho}$$

We normalize $\kappa=r+\rho$ so risk-free debt trades at par, $q^\star=1$. We discuss sovereign risk later.

1.2 Households

There is a continuum of households who differ in their discount rates: some are more patient that others. Household i seeks to maximize

$$\sum_{t=0}^{\infty} \beta_i^t \left(u\left(C_t^i\right) - \kappa_n \frac{N_{i,t}^{1+\varphi}}{1+\varphi} \right), \tag{2}$$

where κ_n is a scaling factor. We index impatient households by b (for borrowers) and patient households by s (for savers). There is a mass χ of b-types and a mass $1-\chi$ of s-types with $\beta_s > \beta_b$. Let $B_{\$,t}^h$ be the nominal face value of the debt issued at t and due at $t+1$, and let $B_t^h \equiv \frac{B_{\$,t}^h}{P_t}$ be the real value of private debt. The *borrowers*' budget constraint is

$$P_t C_t^b = W_t^b N_t^b + P_t \frac{B_t^h}{R_t^h} - P_{t-1} B_{t-1}^h - P_t T_t. \tag{3}$$

subject to a debt constraint

$$B_t^h \leq \bar{B}_t^h.$$

In the equilibria we study below, this constraint ends up binding all the time, making the exact value of the discount factor of borrowers, β_b, matter for welfare but irrelevant for the evolution of the economy. Because of this, we simply write β to denote the saver's discount factor when no confusion results. The *savers*' budget constraint is

$$P_t C_t^s = W_t^s N_t^s + \tilde{R}_t S_{t-1} - S_t - P_t T_t, \tag{4}$$

where $\tilde{R}_t$ is the nominal after-tax gross return on savings S_{t-1}. This return is a complex object since savers hold government bonds, private debt (of borrowers), and equity in corporate businesses. Several assets are traded in our economy. For any asset j that is traded and held by savers, its return must satisfy

$$\mathbb{E}_t \left[\beta \frac{u'\left(C_{t+1}^s\right)}{u'\left(C_t^s\right)} \frac{\tilde{R}_{t+1}^{(j)}}{\Pi_{t+1}} \right] = 1, \tag{5}$$

where $\Pi_{t+1} = P_{t+1}/P_t$ denotes the gross CPI inflation rate from t to $t+1$. Aggregating across types we get

$$\mathbf{C}_t \quad = \quad \chi C_t^b + (1-\chi)\, C_t^s.$$

Finally, under flexible wages, the labor supply condition is

$$\kappa_n N_{i,t}^{\varphi} = \frac{W_{i,t}}{P_t} u'\left(C_t^i\right)$$

for each i. We discuss wage and price rigidities later.

1.3 Production and Market Clearing

Production is linear in labor,

$$Y_t = \mathbf{N}_t - \delta_t \Delta,$$

where δ_t is an indicator of sovereign default, Δ measures the deadweight loss from default, $\mathbf{N}_t$ is an index of labor supplied by borrowers and savers, as in Benigno et al. (2016)

$$\mathbf{N}_t \equiv N_{b,t}^{\chi} N_{s,t}^{1-\chi}. \tag{6}$$

This Cobb-Douglas specification, together with CARA preferences, helps us obtain clear theoretical results. Firms minimize total labor costs $\chi W_{b,t} N_{b,t} + (1-\chi)\, W_{s,t} N_{s,t}$, which implies that per-capita labor income is the same for both types

$$W_{b,t} N_{b,t} = W_{s,t} N_{s,t} = \mathbf{W}_t \mathbf{N}_t,$$

where the wage index is $\mathbf{W}_t \equiv W_{b,t}^{\chi} W_{s,t}^{1-\chi}$. Clearing the goods market requires

$$Y_t = \chi C_t^b + (1-\chi)\, C_t^s + G_t, \tag{7}$$

1.4 Steady-State

We consider a steady state with stable prices, $\Pi = 1$, and $\beta R = 1$. It is convenient to normalize the steady state so that all prices, as well as aggregate consumption, are equal to one. This implies

$$\mathbf{C} = 1$$

$$Y = \mathbf{N} = 1 + G.$$

We then choose the labor supply parameters (κ_n, φ) to support this production level.[2]

2 Simple Example with 2 Periods

We first study a simple model with two periods. The period $t = 1$ is the short run with fixed nominal prices and wages. The period $t = 2$ is the long run with flexible prices and wages. We start with the case without sovereign risk and introduce government default later. The virtue of the simple two-period model is that we obtain quasi-closed form solutions that allow us to understand exactly how sovereign risk affects the model economy.

2.1 Long Run Equilibrium

Let us consider first an equilibrium where the government and the households repay their debts. The budget constraints, assuming no default, are then

$$T_2 = G_2 + \frac{B_1^g}{\Pi_2},$$

$$C_2^b = \frac{\mathbf{W}_2 \mathbf{N}_2}{P_2} - \frac{B_1^h}{\Pi_2} - T_2$$

Savers earn a (possibly random) return from lending to other households and to the government, and they receive dividends from firms. Optimal labor supply implies $\kappa_n N_{i,2}^{\varphi} =$

[2] Assuming for simplicity that production subsidies undo any monopoly distortions, so $P = W$, this requires $\kappa_n (1+G)^{\varphi} = u'(1)$, so we need to set $\kappa_n = \frac{u'(1)}{(1+G)^{\varphi}}$. Note that this is simply a way to scale the steady state to obtain convenient relative prices.

$\frac{W_{i,2}}{\mathbf{P}_2} u'(C_{i,2})$ for each agent and the labor index is defined as $\mathbf{N}_2 = N_{b,2}^{\chi} N_{s,2}^{1-\chi}$. Aggregate consumption is $\mathbf{C}_t = \chi C_t^b + (1-\chi) C_t^s$, and the equilibrium conditions are

$$
\begin{aligned}
&\mathbf{N}_2 = \mathbf{C}_2 + G_2 \\
&C_2^s = \frac{\mathbf{W}_2 \mathbf{N}_2}{P_2} + \frac{\chi}{1-\chi} \frac{B_1^h}{\Pi_2} + \frac{1}{1-\chi} \frac{B_1^g}{\Pi_2} - T_2 + \frac{\left(1 - \frac{W_2}{P_2}\right) Y_2}{1-\chi}.
\end{aligned}
$$

At time 2 we consider a model with flexible (and competitive) wages and prices, so $P_2 = \mathbf{W}_2$. And we use the fact that $W_2^b N_2^b = W_2^s N_2^s = \mathbf{W}_2 \mathbf{N}_2$ to write the equilibrium conditions as

$$
\begin{aligned}
\kappa_n \mathbf{N}_2^{\varphi} &= \left(u'\left(C_2^b\right)\right)^{\chi} \left(u'\left(C_2^s\right)\right)^{1-\chi} \\
C_2^b &= \mathbf{N}_2 - G_2 - \frac{B_1^h + B_1^g}{\Pi_2} \\
C_2^s &= \mathbf{N}_2 - G_2 + \frac{\chi}{1-\chi} \frac{B_1^h + B_1^g}{\Pi_2}
\end{aligned}
$$

2.2 CARA Preferences

We use CARA preferences to obtain closed-form solutions.

$$
u(c) = \frac{-1}{\gamma} \exp(-\gamma c)
$$

Under CARA, we get a simple aggregation result:

$$
\begin{aligned}
\log\left(\kappa_n \mathbf{N}_2^{\varphi}\right) &= -\gamma \left[\chi \left(\mathbf{N}_2 - G_2 - \frac{B_1^h + B_1^g}{\Pi_2}\right) + (1-\chi) \left(\mathbf{N}_2 - G_2 + \frac{\chi}{1-\chi} \frac{B_1^h + B_1^g}{\Pi_2}\right)\right] \\
&= -\gamma \left(\mathbf{N}_2 - G_2\right)
\end{aligned}
$$

In general the equilibrium at time 2 depends on the asset position that each type brings into the period. But, under CARA, these wealth effects between borrowers and savers cancel out exactly. As a result, aggregate labor supply is independent of the distribution of debt balances among households and simply solves

$$\bar{\mathbf{N}}(G) : \log \kappa_n + \varphi \log \bar{\mathbf{N}} = -\gamma \left(\bar{\mathbf{N}} - G \right)$$

In the steady state above, we choose κ_n so that $\bar{\mathbf{N}} = 1 + G$ and therefore $\log \kappa_n = -\gamma - \varphi \log (1 + G)$. Once we have solved for the aggregate, we easily obtain the consumption of each group as

$$C_2^s = 1 + \frac{\chi}{1 - \chi} \frac{B_1^h + B_1^g}{\Pi_2} \tag{8}$$

and

$$C_2^b = 1 - \frac{B_1^h + B_1^g}{\Pi_2}$$

The nice feature of CARA/Cobb-Douglas is a clear dichotomy between the aggregate and the distributional consequences of debt balances. We use this convenient setup in the rest of this section.

2.3 Short Run: Fixed Price Equilibrium

Consider now the equilibrium at time 1 with exogenous prices and wages. The market clearing condition is

$$\mathbf{N}_1 = \mathbf{C}_1 + G_1$$

The government starts with B_0^g debt outstanding and the borrowers with B_0^h so the budget constraints are

$$\frac{B_1^g}{R_1^g} = G_1 - T_1 + \frac{B_0^g}{\Pi_1}$$
$$C_1^b = \mathbf{w}_1 \mathbf{N}_1 + \frac{B_1^h}{R_1^h} - \frac{B_0^h}{\Pi_1} - T_1$$
$$C_1^s = \mathbf{w}_1 \mathbf{N}_1 + \frac{\tilde{R}_1}{\Pi_1} S_0 - S_1 - T_1 + \frac{(1 - \mathbf{w}_1) Y_1}{1 - \chi}$$

where $\mathbf{w}_1$ is the real wage, $\tilde{R}_1$ is the nominal rate of return earned by savers, who also receive dividends from firms $(1-\mathbf{w}_1)\, Y_1$. Borrowers are subject to the borrowing limit

$$B_1^h \leq \bar{B}_1^h$$

Prices and wages are exogenous at time 1 and we ignore the labor supply curves. The savers' Euler equation is

$$\mathbb{E}_1 \left[\beta \frac{u'\left(C_2^s\right)}{u'\left(C_1^s\right)} \frac{\tilde{R}_2}{\Pi_2} \right] = 1$$

where $\tilde{R}_2$ is the nominal return earned by savers at time 2. The return can be random if there is credit risk and/or aggregate uncertainty. In this section, however, we consider the case where all debts are risk free so R_1 is the same for all households and for the government, and therefore we have $R_1 = \tilde{R}_2$. We normalize $\Pi_1 = 1$. The equilibrium conditions become

$$\begin{aligned}
\frac{B_1^g}{R_1^g} &= G_1 - T_1 + B_0^g \\
(1-\chi)\, S_1 &= \frac{B_1^g}{R_1^g} + \chi \frac{B_1^h}{R_1^h} \\
(1-\chi)\, \tilde{R}_1 S_0 &= B_0^g + \chi B_0^h \\
C_1^s &= \mathbf{w}_1 \mathbf{N}_1 + \tilde{R}_1 S_0 - S_1 - T_1 + \frac{1-\mathbf{w}_1}{1-\chi} Y_1
\end{aligned}$$

The government chooses T_1 and the private debt limit is exogenous $\bar{B}_1^h$. Using market clearing at time 1, we can solve for the equilibrium as a function of R_1 and T_1. Equilibrium in financial market at time 1 requires

$$\tilde{R}_1 S_0 - S_1 = \frac{1}{1-\chi} \left(B_0^g - \frac{B_1^g}{R_1^g} \right) + \frac{\chi}{1-\chi} \left(B_0^h - \frac{B_1^h}{R_1^h} \right)$$

which then implies

$$C_1^s = \left(\mathbf{w}_1 + \frac{1-\mathbf{w}_1}{1-\chi} \right) \mathbf{N}_1 - G_1 - \frac{\chi}{1-\chi} \left(\frac{B_1^g}{R_1^g} - B_0^g + \frac{\bar{B}_1^h}{R_1^h} - B_0^h \right) \tag{9}$$

this gives us C_1^s as a function of $\mathbf{N}_1$ and exogenous driving forces and pre-determined variables. The first two terms of the equations capture the classic Ricardian terms: savers earn labor income and receive dividends, and they pay for government spending G_1. The last term is the non-Ricardian one. Savers must finance net lending to the government $\frac{B_1^g}{R_1^g} - B_0^g = G_1 - T_1$ and to the private sector $\frac{\bar{B}_1^h}{R_1^h} - B_0^h$. Ricardian equivalence holds when $\chi = 0$, in which case T_1 does not matter for $\mathbf{C}_1$. Otherwise, an increase in T_1 decreases the consumption of impatient agents, and given $\mathbf{N}_1$, it must increase the consumption of savers.

2.4 Equilibrium without default

The link between the two periods comes from the Euler equation

$$u'(C_1^s) = \beta \frac{R_1}{\Pi_2} u'(C_2^s) \tag{10}$$

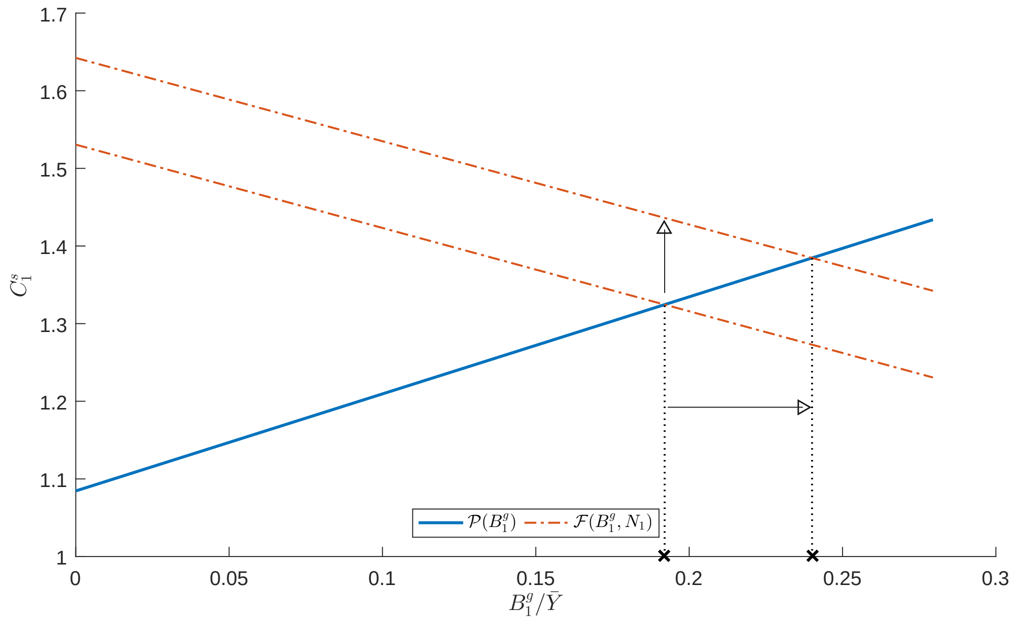
Without default risk we have $R_1^g = R_1$ and we can write (8) as

$$C_2^s = 1 + \frac{\chi}{1-\chi} \frac{\bar{B}_1^h + B_1^g}{\Pi_2} \tag{11}$$

The equilibrium is characterized by equations (9,10,11) together with a specification of inflation and monetary policy. Consistent with our assumption of a small (closed) economy in a currency union, we consider the case $\Pi_2 = 1$ and $\beta R_1 = 1$.[3] This equilibrium is depicted in Figure 1.

[3] The same equations can also be used to think about a closed economy with independent monetary policy. For instance, we can look for the policies that implement $\mathbf{N}_1 = \bar{N}(G_1)$: given T_1 and Π_2, the monetary policy rate R_1. Alternatively, we can consider an economy in a liquidity trap at time 1, $R_1 = 1$. We can think about forward guidance and commitment to a future Π_2. Or we can assume no commitment, normalize $\Pi_2 = 1$, and consider the equilibrium as a function of T_1.

Figure 1: Equilibrium without default risk



Note: the arrows measure the change in $\mathbf{N}_1$ given the change in B_1^g assuming $\mathbf{w}_1 = 1$

We can describe the equilibrium with two curves. The financial wealth curve comes from the Euler equation (10) and the equilibrium budget constraint (11) of the savers. It describes a schedule $C_1^s = \mathcal{P}(B_1^g)$ which is increasing in B_1^g:

$$\mathcal{P}(B_1^g) \equiv 1 + \frac{\chi}{1-\chi}\left(\bar{B}_1^h + B_1^g\right). \tag{12}$$

The funding curve $C_1^s = \mathcal{F}(B_1^g; \mathbf{N}_1)$ is simply equation (9) and it describes a schedule which is decreasing in B_1^g and increasing in $\mathbf{N}_1$

$$\mathcal{F}(B_1^g; \mathbf{N}_1) \equiv \left(\mathbf{w}_1 + \frac{1-\mathbf{w}_1}{1-\chi}\right)\mathbf{N}_1 - G_1 - \frac{\chi}{1-\chi}\left(\frac{B_1^g + \bar{B}_1^h}{R_1} - B_0^g - B_0^h\right) \tag{13}$$

Proposition 1. *The equilibrium without sovereign risk is given by* $\mathbf{N}_1(B_1^g)$ *such that*

$$\mathcal{P}(B_1^g) = \mathcal{F}(B_1^g; \mathbf{N}_1).$$

Employment increases with $\left(G_1, \mathbf{w}_1, B_1^g + B_1^h\right)$ *and decreases with* $B_0^g + B_0^h$. *Private and public* *debt are perfect substitutes for aggregate demand with a multiplier equal to* $\frac{\chi}{1-\chi}(1+\beta)$.

Note that in the simple model considered here we can obtain a closed form solution for $\mathbf{N}_1$ as a function of B_1^g:

$$\left(1+\frac{\chi}{1-\chi}(1-\mathbf{w}_1)\right)\mathbf{N}_1 = 1+G_1+\frac{\chi}{1-\chi}\left((1+\beta)\left(\bar{B}_1^h+B_1^g\right)-B_0^g-B_0^h\right) \tag{14}$$

We have the neoclassical terms first, then the non Ricardian terms that depend on $\chi > 0$. The multiplier on government debt is $\frac{\chi}{1-\chi}(1+\beta)$. The term $\frac{\chi}{1-\chi}$ is the fundamental non-Ricardian factor. But because it appears both in the wealth equation (12), and in the funding equation (13) the total multiplier is $1+\beta$ times the non-Ricardian factor.

To understand the welfare analysis, it is important to note that all debts will be repaid, if not in period 1, then in period 2. A natural benchmark is then to repay roughly half in the first period, and half in the second period. More precisely, if $\bar{B}_1^h = \frac{B_0^h}{1+\beta}$ and $B_1^g = \frac{B_0^g}{1+\beta}$, and if $\mathbf{w}_1 = 1$ then we have $\mathbf{N}_1 = 1+G_1$, which is the natural rate of employment.

Lemma 1. *The natural rate of employment is achieved when real wages are at their long run equilibrium* $\mathbf{w}_1 = 1$ *and private and public debt repayment are balanced over time* $\bar{B}_1^h = \frac{B_0^h}{1+\beta}$ *and* $B_1^g = \frac{B_0^g}{1+\beta}$.

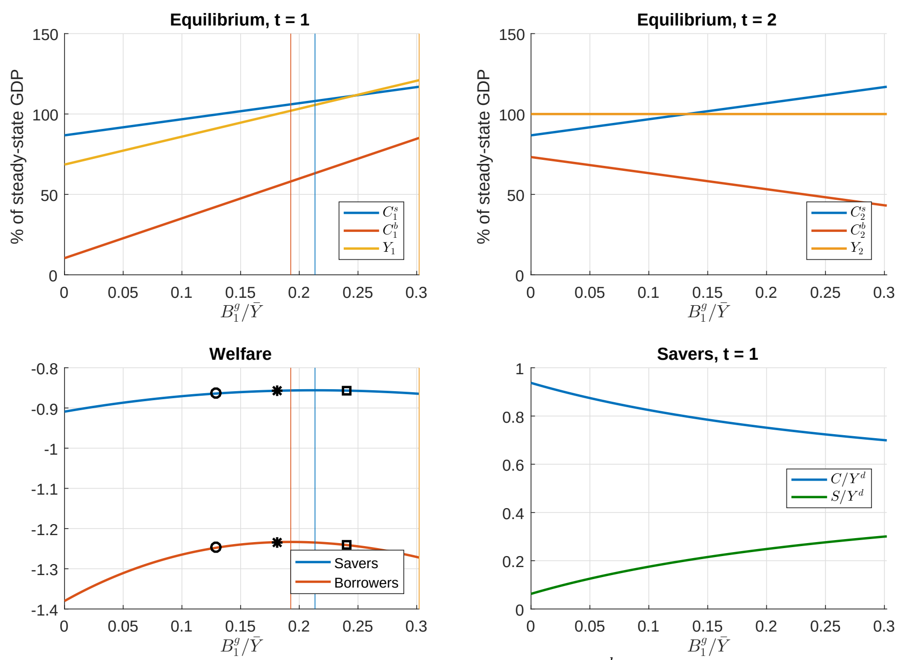
Equation (14) also allows us to change the state of the economy. We can create a demand-driven recession with private deleveraging or with low real wages. The economy can be depressed when $(1+\beta)\bar{B}_1^h - B_0^h < 0$ because this affects the consumption of constrained agents.[4] Figure 2 analyzes fiscal policy in a recession induced by private deleveraging, i.e., created by an excessively quick repayment of private debts: $\bar{B}_1^h < \frac{B_0^h}{1+\beta}$. Savers prefer a point near full employment, i.e., a point where the government compensates private deleveraging with expansionary fiscal policy in such a way that the economy achieves (roughly) a balanced debt repayment $\bar{B}_1^h + B_1^g \approx \frac{B_0^g+B_0^h}{1+\beta}$. This is not exactly true because of various second order

[4]Low real wages (or high profits $1-\mathbf{w}_1$) also depress the economy because the savers earn the profits but have a smaller propensity to consume than the borrowers. In the case $\bar{B}_1^h = \frac{B_0^h}{1+\beta}$ and $\mathbf{w}_1 < 1$, our results were very similar to the ones shown for the deleveraging recession.

distributional effects (savers understand that more expansionary policy increases their total wealth, and they are willing to finance the government's deficits today in exchange for more lifetime consumption), but it clearly captures the main preference of the savers.

The relative impatience of borrowers makes them less willing to finance the government even when this stimulates the economy. This makes them prefer a point closer to full employment than savers do. Both types then disagree about the optimal path but the differences are not dramatic. The dynamic case will tell a different story.

Figure 2: Private Deleveraging Recession without Sovereign Risk



Note: Natural rate of output is 100. Recession is driven by $\bar{B}_1^h < \frac{B_0^h}{1+\beta}$. Black circles correspond to neutral policy $B_1^g = \frac{B_0^g}{1+\beta}$. Black stars correspond to full employment in period 1. Black squares correspond to taxes that keep debt constant. Vertical bars mark welfare-maximizing policies for each group.

2.5 Sovereign Risk

Let us now introduce sovereign risk. The government can default between time 1 and time 2 and the risk of default increases with the debt burden B_1^g.[5] Let δ be a indicator of default. We assume that the probability of default is given exogenously by the increasing function

[5] Equivalently, we could normalize by GDP or we could limit how much the government can tax $T_2 = G_2 + \frac{B_1^g}{\Pi_2}$ at time 2.

$\pi(\cdot)$:

$$\Pr(\delta = 1) = \pi\left(B_1^g; \epsilon\right),$$

where we think of ϵ as an exogenous shifter of credit risk which is useful for comparative statics. Note that this assumption is not standard in the sovereign debt literature, where default is usually a choice for the government. The simplification here allows us to focus on the endogenous pricing of debt by domestic savers instead. In this model, the probability of default is exogenous (given the current stock of debt) but the risk premium is endogenous.

In case of default, the government imposes a haircut $\hbar$ and repays only $(1-\hbar) B_1^g$. In addition, we introduce a deadweight loss to output of Δ (which may be zero), which changes the market clearing condition as

$$\mathbf{N}_2 - \delta\Delta = \mathbf{C}_2 + G_2.$$

We can solve for the equilibrium labor supply at time 2 as a function of the occurrence of default

$$\mathbf{N}_2 = \bar{\mathbf{N}}(\delta) : \ \log \kappa_n + \varphi \log \mathbf{N} = -\gamma\left(\mathbf{N} - G_2 - \delta\Delta\right)$$

and G_2 is fixed so we drop it from the list of arguments, and as before we normalize the preferences so that $\bar{\mathbf{N}}(0) = 1 + G$. The important point is that redistributive shocks do not affect the aggregate labor index, so $\mathbf{N}$ does not depend on $\hbar$. The consumption of savers is random for two reasons, the deadweight loss Δ and the haircut $\hbar$:

$$C_2^s(\delta) = \bar{\mathbf{N}}(\delta) - G_2 - \delta\Delta + \frac{\chi}{1-\chi}\frac{B_1^h + (1-\delta\hbar) B_1^g}{\Pi_2}$$

At time 1 the savers understand that sovereign debt is risky, which induces *precautionary savings*. Savers have a portfolio. They can save risk free at rate R_1, either abroad or by lending to borrowers. Their Euler equation implies

$$u'\left(C_1^s\right) = \mathbb{E}_1\left[\frac{\beta R_1}{\Pi_2} u'\left(C_2^s\right)\right]$$

We consider a small economy with exogenous monetary policy, and we set $\beta R_1 = 1$ and $\Pi_2 = 1$. The Euler equation becomes

$$
\begin{aligned}
-\gamma C_1^s &= \log\left((1-\pi)\, e^{-\gamma C_2^s(0)} + \pi e^{-\gamma C_2^s(1)}\right) \\
&= \log\left((1-\pi)\, e^{-\gamma\left(\bar{N}(0) - G_2 + \frac{\chi}{1-\chi}\left(B_1^h + B_1^g\right)\right)} + \pi e^{-\gamma\left(\bar{N}(1) - G_2 - \Delta + \frac{\chi}{1-\chi}\left(B_1^h + (1-\hbar) B_1^g\right)\right)}\right)
\end{aligned}
$$

This defines a new wealth function $C_1^s = \mathcal{P}\left(B_1^g; \pi\right)$ which is increasing in B_1^g and decreasing in π

$$
\mathcal{P}\left(B_1^g; \pi\right) \equiv 1 + \frac{\chi}{1-\chi}\left(B_1^h + B_1^g\right) - \frac{1}{\gamma}\log\left(1 - \pi + \pi e^{\gamma\left(\bar{N}(0) + \Delta - \bar{N}(1) + \frac{\chi}{1-\chi}\hbar B_1^g\right)}\right) \tag{15}
$$

Note that $\bar{N}(0) + \Delta - \bar{N}(1) + \frac{\chi}{1-\chi}\hbar B_1^g > 0$, so $\log\left(1 - \pi + \pi e^{\gamma\left(\bar{N}(0) + \Delta - \bar{N}(1) + \hbar\frac{\chi}{1-\chi} B_1^g\right)}\right)$ is increasing in π. If we specify the schedule $\pi\left(B_1^g; \epsilon\right)$ we can then solve for

$$
C_1^s = \mathcal{P}\left(B_1^g; \pi\left(B_1^g; \epsilon\right)\right)
$$

which is decreasing in ϵ. The schedule as a function of B_1^g is both lower and flatter than before because of the default risk. The direct multiplier is still $\frac{\chi}{1-\chi}$ but an increase in B_1^g has two other effects via credit risk. For a given π it increases the losses in the bad state $\frac{\chi}{1-\chi}\hbar B_1^g$. It also increases π. Both effects lower the value of debt and therefore consumption. If these effects are very strong (π a step function for instance), then it is possible for the schedule $\mathcal{P}\left(B_1^g; \pi\left(B_1^g; \epsilon\right)\right)$ to be *decreasing* in B_1^g, at least locally.[6]

The funding constraint $C_1^s = \mathcal{F}\left(B_1^g, \epsilon; \mathbf{N}_1\right)$ is

$$
\mathcal{F}\left(B_1^g, \epsilon; \mathbf{N}_1\right) \equiv \left(\mathbf{w}_1 + \frac{1-\mathbf{w}_1}{1-\chi}\right)\mathbf{N}_1 - G_1 - \frac{\chi}{1-\chi}\left(q\left(B_1^g; \epsilon\right)\frac{B_1^g}{R_1} - B_0^g + \frac{\bar{B}_1^h}{R_1} - B_0^h\right) \tag{16}
$$

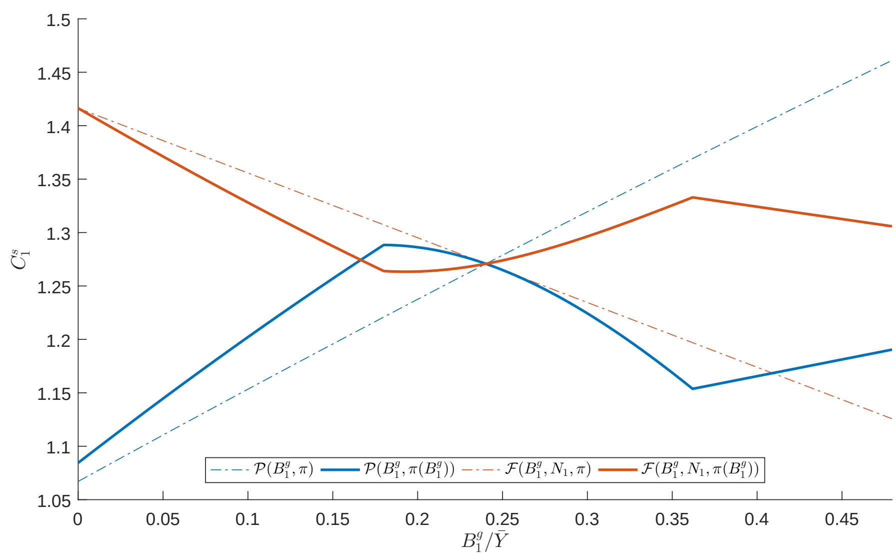
[6] Notice that γ appears twice in the last term of equation (15). The first time, $\frac{1}{\gamma}$ reduces the impact of sovereign risk, as higher values of γ make savers more willing to substitute consumption between periods. The second time, γ amplifies the difference in output between states, reflecting the fact that higher risk aversion induces more savings in the fact of variance.

where the price of government bonds $q\left(B_1^g;\epsilon\right)$ is determined by savers as

$$\begin{aligned} q_1 &= \mathbb{E}_1\left[\beta\frac{u'\left(C_2^s\right)}{u'\left(C_1^s\right)}\left(1-\delta\hbar\right)\right] \\ &= \frac{1}{R_1}-\hbar\mathbb{E}_1\left[\beta\frac{u'\left(C_2^s\right)}{u'\left(C_1^s\right)}\delta\right] \\ &= \frac{1}{R_1}-\beta\hbar\pi\left(B_1^g;\epsilon\right)e^{\gamma\left(C_1^s-C_2^s(1)\right)} \end{aligned}$$

We can see that $q\left(B_1^g;\epsilon\right)$ is decreasing in both arguments. As a result, the funding schedule (16) is increasing in ϵ and less steep as a function of B_1^g than before. Again, if the price effect is strong, we can get the funding curve to be locally *increasing* in B_1^g.

Figure 3: Equilibrium with Sovereign Default Risk



Note: Equilibrium is unique conditional on a choice of B^g. But three choices (one of them with a higher level of debt and not depicted) of B^g are consistent with the same level of employment $\mathbf{N}_1$.

An equilibrium must satisfy

$$\mathcal{P}\left(B_1^g;\pi\left(B_1^g;\epsilon\right)\right)=\mathcal{F}\left(B_1^g,\epsilon;\mathbf{N}_1\right)$$

Figure 3 shows the equilibrium. The straight dashed lines are drawn for fixed π, equal to the equilibrium value. This is the case where credit risk is not responsive to leverage. The solid lines correspond to the equilibrium pricing function calibrated below. Note that our timing convention implies a unique equilibrium. Uniqueness obtains because we assume that the government chooses B_1^g and then that the markets price the bonds knowing B_1^g and therefore, implicitly, that the government stands ready to adjust taxes to obtain B_1^g for any price. The alternative timing/commitment assumption of Calvo (1988) can yield multiple equilibria (see Lorenzoni and Werning, 2013, for a discussion).

Proposition 2. *The equilibrium with sovereign risk is given by* $\mathbf{N}_1 \left(B_1^g, \epsilon\right)$ *such that*

$$\mathcal{P}\left(B_1^g; \pi\left(B_1^g; \epsilon\right)\right) = \mathcal{F}\left(B_1^g, \epsilon; \mathbf{N}_1\right)$$

Employment increases with $\left(G_1, \mathbf{w}_1, B_1^h\right)$ *and decreases with* $B_0^g + B_0^h$*. Private and public debt are no longer perfect substitutes and the sovereign multiplier is smaller. Employment may decrease with* B_1^g *when* $\pi\left(\cdot\right)$ *is steep enough.*

But Figure 3 makes clear that there are strong complementarities in the model. More precisely, the figure shows that there are three levels of B_1^g that are consistent with the same output in the first period. Aggregate efficiency is of course higher when debt is lower, because, for given $\mathbf{N}_1$, lower debt reduces default risk and expected deadweight losses. This is not a Pareto-improvement per-se because the borrowers might prefer default and lower taxes. To make it Pareto superior we would need to let the government adjust relative transfers at time 2 based upon whether default occurs or not.

2.6 Calibration

Figure 4 summarizes our results when we introduce sovereign risk.[7] The critical feature added here is the π function which describes the probability of default. To calibrate it, we estimate an equation for average sovereign spreads during the eurozone crisis using data from Martin and Philippon (2017). Martin and Philippon (2017) estimate a non-linear mapping between spreads and sovereign debt. Each country's spread depends on its own sovereign debt, with a piecewise linear schedule. The loadings are time varying according to an aggregate factor. Using their data we can estimate

[7]The panel marked $T = 2$ now includes output and consumption when default happens, in dashed lines.

Period	Mapping
Crisis Times (Europe 2011-2012)	$Spread_t^{crisis} = 0.01 \cdot \mathbf{1}\left(B_{t-2}^g \leq 0.9\right) B_{t-2}^g + 0.1 \cdot \mathbf{1}\left(B_{t-2}^g > 0.9\right)\left(B_{t-2}^g - 0.9\right)$
"Normal" Times (Europe post 2013)	$Spread_t^{normal} = 0.003 \cdot \mathbf{1}\left(B_{t-2}^g \leq 0.9\right) B_{t-2}^g + 0.035 \cdot \mathbf{1}\left(B_{t-2}^g > 0.9\right)\left(B_{t-2}^g - 0.9\right)$

Table 1: Estimated Mappings, based on Martin and Philippon (2017)

Notes: $Spread_t$ is the annual spread over the German interest rate, and B_t^g is government debt rebased by potential GDP

These numbers imply an essentially flat default probability until debt reaches 90% of GDP. On the other hand, when debt/GDP is around 1, an increase of debt of 10% of GDP would move spreads by around 1%. For instance, during the crisis, the Italian debt to GDP ratio was around 1.3. During the crisis period, the spread of long term Italian bonds over long term German bonds was around 4% (the average German yield was 1.5% in 2012 compared to 5.5% for Italy). This is consistent with an elasticity of spreads to debt of $4\%/\left(1.3-0.9\right)=0.1$

According to the OECD, the average duration of Central Government Debt for Italy is around 5 years (4.9 years in 2010). We assume a risk-adjusted loss rate in case of default of $\hbar = 0.5$. This gives us a π function of the form:

$$\pi^{crisis} = \frac{5}{0.5}\left(0.01 \cdot \frac{B_1^g}{\bar{Y}/5} + (0.1-0.01)\left(\frac{B_1^g}{\bar{Y}/5} - 0.9\right)\mathbf{1}\left(\frac{B_1^g}{\bar{Y}/5} > 0.9\right)\right).$$

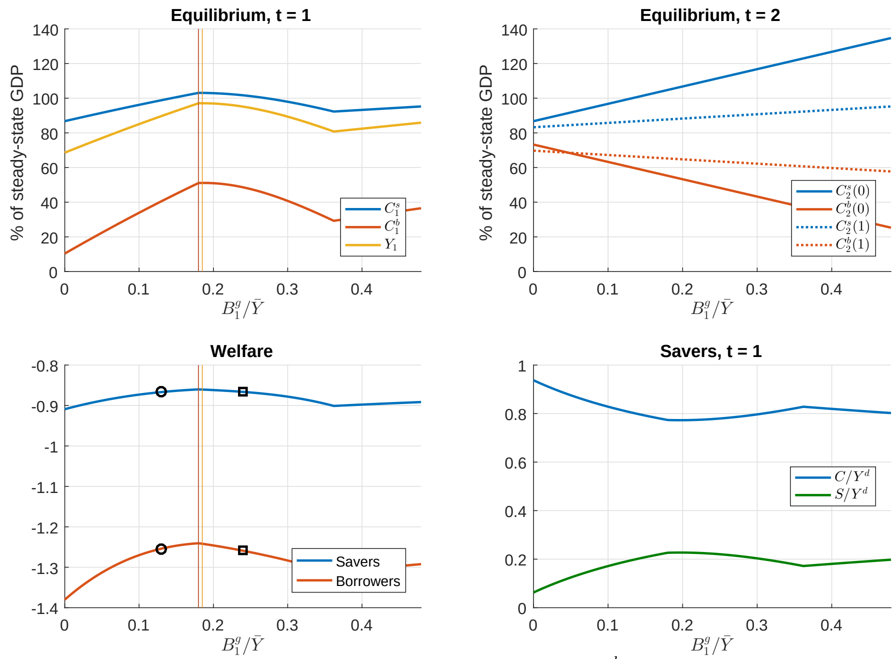
Since the "whatever it takes" speech by Mario Draghi in the summer of 2012, the spreads have been divided (roughly) by 3, as one can see from the Table above. Therefore we set

$$\pi^{normal} = \frac{1}{3}\pi^{crisis}.$$

We will explore the implications of both specifications.

Full employment in the short run is no longer feasible in this economy. In the long run if debt is zero, the cost of default shows up as lower consumption for all; when debt is positive, default is rather a redistributive tool. Figure 4 reveals some new dynamics: borrowers want to maximize output at time 1. They understand the recessionary effects of sovereign risk, so they support some debt repayments in the first period, despite the recession, to the extent

Figure 4: Deleveraging with Sovereign Risk



Note: Natural rate of output is 100. Recession is driven by $\bar{B}_1^h < \frac{B_0^h}{1+\beta}$. Full employment in period 1 is no longer feasible. Black squares correspond to taxes that keep debt constant. Vertical bars mark welfare-maximizing policies for each group.

that these prove effective in reducing the probability of default. In the steep region of the π function, moreover, tax increases are expansionary. The reason why this happens can be linked to the precautionary behavior of savers. Indeed, in the critical intermediate region, the savers exhibit a much higher marginal propensity to consume, as they expect the high consumption and no default state to happen with ever higher probability. Therefore, when the government raises taxes in this region, the savers' consumption response more than compensates for the borrowers' spending cuts.

3 Dynamic Risk-Sensitive Economy

We consider a model with an infinite horizon but truncated in the sense that after some (large) $\mathcal{T}$ we assume that the economy is in its flexible price steady state without default risk. To compute the solution we start from period $\mathcal{T}$. In all periods $t < \mathcal{T}$, we assume that prices, wages, and the nominal (risk free) interest rate are fixed. The model is calibrated at quarterly frequency (one period is one quarter). We now assume that households' preferences are

$$V_t^{\frac{\psi-1}{\psi}} = (1-\beta)\, u_t^{\frac{\psi-1}{\psi}} + \beta \left(\mathbb{E}_t \left[V_{t+1}^{1-\gamma}\right]\right)^{\frac{\psi-1}{\psi(1-\gamma)}}, \tag{17}$$

$$u_t = C_t^{\alpha} \left(\kappa_n - N_t\right)^{1-\alpha}$$

where ψ is the elasticity of inter-temporal substitution and γ is the risk aversion parameter. This assumption allows us to disentangle the effects of risk aversion and intertemporal substitution already emphasized in equation (15). κ_n and α are normalized to obtain the same steady-state as in section 1.4. The savers' pricing kernel is

$$M_{t+1} = \beta \left(\frac{C_{t+1}^s}{C_t^s}\right)^{-1} \left(\frac{u_{t+1}^s}{u_t^s}\right)^{\frac{\psi-1}{\psi}} \left(\frac{V_{s,t+1}}{\mathbb{E}_t \left[V_{s,t+1}^{1-\gamma}\right]^{\frac{1}{1-\gamma}}}\right)^{\frac{1}{\psi}-\gamma}. \tag{18}$$

This is the pricing kernel that is going to price government debt in our model.

3.1 Government Default

The government can default once (and only once) at any time $t < \mathcal{T}$ and the risk of default increases with the debt burden B_t^g, as before. Let $\mathcal{H}_t$ be the history of default up to time t: $\mathcal{H}_t = 0$ if and only if there has been no default up to and including time t. Note that our earlier assumption that risk disappears after $\mathcal{T}$ is simply that $\Pr\left(\mathcal{H}_s = 0 \mid \mathcal{H}_T = 0\right) = 1$ for all s. In case of default, the government imposes a haircut $\hbar$ and its budget constraint becomes

$$T_t^d + q_t^d \left(B_t^{g,d} - (1-\hbar)(1-\rho)\, B_{t-1}^g\right) = G_t + (1-\hbar)\, \kappa B_{t-1}^g$$

where q_t^d and $B_t^{g,d}$ are the price and the amount of new debt after default, and T_t^d is the level of taxes after default. In addition, default creates a transitory deadweight loss of output Δ so the resource constraint is

$$\mathbf{N}_t \left(1 - \mathbf{1}_{(\mathcal{H}_t=1)}\Delta\right) = C_t + G_t$$

where $\mathbf{1}_x$ is an indicator function. After a default has happened, the economy exits the low productivity state with constant hazard θ. When this happens, we write $\mathcal{H}_t = 2$ to indicate the post-default state where productivity has returned to normal. Debt is still affected by the haircut.

3.2 Long Run Equilibrium

For $t \geq \mathcal{T}$, there is no default risk so the price of government debt is $q^\star = 1$. The government keeps the level of debt constant so taxes are $T = G + rB^g$. Without CARA preferences, the distribution of wealth affects aggregate labor supply, so we assume that the government uses time-invariant transfers T_s and T_b to achieve $C^s = C^b$ in the initial steady-state.[8] The borrower's budget constraint is

$$C_t^b = \frac{W_t^b}{P_t} N_t^b + \frac{B_t^h}{R_t^h} - \frac{B_{t-1}^h}{\Pi_t} - T_t + T_b.$$

In steady state we have

$$C^b = \frac{W^b N^b}{P} - \frac{r}{1+r} B^h - T + T_b.$$

The net payment of each borrower is $\frac{r}{1+r} B^h$. Adapting our earlier analysis we know that savers will consume

$$C_{\mathcal{T}}^s \left(\mathcal{H}_{\mathcal{T}}\right) = \left(1 - \mathbf{\Delta 1}_{(\mathcal{H}_{\mathcal{T}}=1)}\right) \mathbf{N}\left(\mathcal{H}_{\mathcal{T}}\right) - G + r \frac{\chi}{1-\chi} \left(\frac{B_{\mathcal{T}-1}^h}{1+r} + B_{\mathcal{T}-1}^g \right) + T_s$$

We choose the parameter κ_n to normalize aggregate consumption without default: $\mathbf{C}(0) = 1$ and $\mathbf{N}(0) = 1 + G$ in the initial flexible price equilibrium.

In the flexible price equilibrium that ensues for $t \geq \mathcal{T}$, we proceed by first guessing the aggregate labor supply $\mathbf{N}_{\mathcal{T}}\left(\mathcal{H}_{\mathcal{T}}\right)$ and deducing consumption of savers and borrowers from their budget constraints.[9] With this, we have two equations for each type to determine labor supply

[8] We impose $\chi T_b + (1-\chi) T_s = 0$ so these transfers are self-financing. This assumption is only introduced for tractability, to ensure that the steady-state distribution of wages is independent of the distribution of income. These transfers represent 5.41% of GDP in our simulations.

[9] Under flexible wages, the wage index is $\mathbf{W}_{\mathcal{T}} = 1$, firms make zero profits, and consequently income of both types is $W_{\mathcal{T}}^i N_{\mathcal{T}}^i = \mathbf{W}_{\mathcal{T}} \mathbf{N}_{\mathcal{T}}$.

and wages for that type: the firm's first-order condition to hire each type $W^i_{\mathcal{T}} N^i_{\mathcal{T}} = \mathbf{W}_{\mathcal{T}} \mathbf{N}_{\mathcal{T}}$ along with each type's first-order condition to supply labor $N^i_{\mathcal{T}} = \kappa_n - \frac{1-\alpha}{\alpha} \frac{C^i_{\mathcal{T}}}{W^i_{\mathcal{T}}}$. We then iterate over the guess until $\mathbf{N}_{\mathcal{T}} = \chi N^b_{\mathcal{T}} + (1-\chi) N^s_{\mathcal{T}}$.

Definition Given a path for the private borrowing limit $\left\{\bar{B}^h_t\right\}_{t=0}^{\mathcal{T}}$, a path for government debt $\{B^g_t\}_{t=0}^{\mathcal{T}}$, and wages $\left\{W^s, W^b\right\}$, a *competitive equilibrium* consists of an allocation of consumption and labor supply $\left\{C^s_t(\mathcal{H}_t), C^b_t(\mathcal{H}_t), N^s_t(\mathcal{H}_t), N^b_t(\mathcal{H}_t)\right\}_{t=0}^{\mathcal{T}}$, along with taxes $\{T_t(\mathcal{H}_t)\}_{t=0}^{\mathcal{T}}$ and prices of government debt $\{q_t(\mathcal{H}_t)\}_{t=0}^{\mathcal{T}}$, such that

1. Given taxes and prices, the consumption allocation is consistent with individual maximization (ie, the Euler equation for savers and the budget constraint for borrowers), where the steady-state described above is reached in period $\mathcal{T}$.

2. Taxes and the price of debt satisfy the government's budget constraint given the path of debt,

3. Markets clear

3.3 Dynamics

We take the path of private debt limits $\bar{B}^h_t$ as exogenous and use it to engineer a demand-driven recession in the economy, similar to the recessions that have been observed in many countries in 2008-2009. In the benchmark model, the path of private deleveraging is independent of fiscal policy and of government default. Wages are fixed until period $\mathcal{T}$ when they become flexible again. We then need to specify fiscal policy, and we want to think of a situation where the government leaves the tax rate constant until it starts deleveraging.[10] This way, sovereign debt starts accumulating if its price drops, or if the economy enters a recession.

The fiscal tradeoff is clear. The government could wait until the recession is over to collect the extra taxes it needs to pay the debt. In that case the presence of default risk pushes down

[10]In the model, taxes are lump-sum. However, we believe that we should still take into account the fact that tax revenues drop in recessions. Moreover, under rigid wages, our model is identical to one in which the government only collects a proportional labor income tax. We prefer the lump-sum taxes formulation to make it clear that labor supply decisions are not affected.

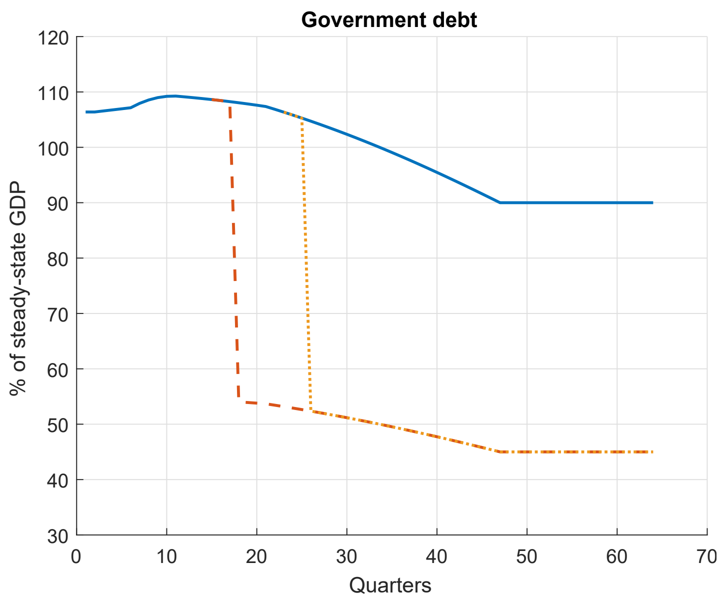
the price of debt which accelerates debt accumulation, while the precautionary behavior of savers drains aggregate demand.

We finally assume that, in case of default, debt is reduced by the haircut $\hbar$ but the path of debt dynamics does not change. In other words, $B_t^{g,2} = B_t^{g,1} = (1-\hbar)\, B_t^{g,0}$. The actual path is

$$B_t^g\left(\mathcal{H}_t\right) = \left(1-\mathcal{H}_t\right) B_t^{g,0} + \mathcal{H}_t B_t^{g,1}.$$

Figure 5 shows a path of public deleveraging with and without default.

Figure 5: Sample Paths of Public Debt with and without Default



Note: Solid line is path without default. Dashed and dotted lines are two examples of default paths.

Consistent with our definition of equilibrium, we solve the model by backward induction starting from the final steady-state. Our assumptions about haircuts, deadweight losses and, most importantly, the path of sovereign debt being affected by default in a homothetic way imply that the equilibrium path after default does not depend on when default took place.

Lemma 2. *Let* $\mathbf{N}_t^{\mathcal{H}_t,t'}$ *denote employment at time* t *in an economy where default occurred at time* $t' < t$ *and the current default state is* $\mathcal{H}_t \in \{1,2\}$. *Then for all* (t', t'') *we have*

$$\mathbf{N}_t^{\mathcal{H}_t,t'} = \mathbf{N}_t^{\mathcal{H}_t,t''}$$

As a result we only need to solve for one default path. Notice also that, as default can happen only once, we have that $q_t^2 = q_t^1 = q^\star = 1$ and, consequently, the government's budget constraint is

$$T_t^{\mathcal{H}_t} + q_t^{\mathcal{H}_t}\left(1 - \mathbf{1}_{(\mathcal{H}_t=1)}\hbar\right)\left(B_t^{g,0} - (1-\rho)\,B_{t-1}^{g,0}\right) = G + (1-\hbar)\,\kappa B_{t-1}^{g,0}.$$

3.3.1 Backward induction

At period $\mathcal{T}$, we can solve for the equilibrium with flexible wages and normal TFP,[11] as discussed above. Because we proceed by backward induction, at any time t we know the state of the economy at $t+1$. Specifically, we know the stock of debt B_t^g that the government needs to repay at $t+1$, which is what we need to compute the default probability $\pi_t = \pi\left(B_t^g\right)$. Because we also know C_{t+1}^s in both the default and no default states and given $\beta R = 1$, we can use the stochastic discount factor (18) into the Euler equation for risk-free debt

$$\begin{aligned}
(1-\pi_t)\,M_{t+1}^{0,0} + \pi_t M_{t+1}^{0,1} &= \beta \\
(1-\theta)\,M_{t+1}^{1,1} + \theta M_{t+1}^{1,2} &= \beta \\
M_{t+1}^{2,2} &= \beta
\end{aligned}$$

where $M_{t+1}^{\mathcal{H}_t,\mathcal{H}_{t+1}}$ is the sdf of savers between states $\mathcal{H}_t$ and $\mathcal{H}_{t+1}$. Under expected utility, this Euler equation reduces to an equation in future variables and current consumption. Here, current-period terms depend on current consumption and the current flow of utility (i.e., the current labor supply), so we only get the product $C_{s,t} \cdot u_{s,t}^{\frac{\psi-1}{\psi}}$ (for each state). Nevertheless, that is enough to compute the values of $M_{t+1}^{0,0}$, $M_{t+1}^{0,1}$, $M_{t+1}^{1,1}$, $M_{t+1}^{1,2}$, and $M_{t+1}^{2,2}$ separately. With the stochastic discount factor in hand, we can move to the pricing of undefaulted government debt,

$$q_t^0 = (1-\pi_t)\,M_{t+1}^{0,0}\left(\kappa + (1-\rho)\,q_{t+1}^0\right) + \pi_t M_{t+1}^{0,1}\,(1-\hbar)\,(\kappa + (1-\rho)\,q^\star)\,.$$

[11] We assume that when the economy reaches $\mathcal{T}$, TFP goes back to normal even in the unlikely event that the θ shock has not yet realized. Our results are unaffected by this choice.

If we knew the stock of debt outstanding $B_{t-1}^{g,0}$, as we do when deleveraging has already ended some time before t, we could look at the budget constraint of the government to deduce what taxes are. In general, however, we only know what the tax rate $\frac{T_t^0}{Y_t^0}$ should be. Note that the tax rate has current output in it, which makes it an endogenous object. So we proceed by guessing the stock of debt $B_{t-1}^{g,0}$, computing the values of the endogenous variables for period t, and finally iterating over this guess until we get the desired tax rate. Given $B_{t-1}^{g,0}$, the budget constraint of the government gives us tax collections T_t as

$$T_t^0 + q_t^0 \left(B_t^{g,0} - (1-\rho) B_{t-1}^{g,0} \right) = G + \kappa B_{t-1}^{g,0}.$$
$$T_t^1 + (1-\hbar) \left(B_t^{g,0} - (1-\rho) B_{t-1}^{g,0} \right) = G + (1-\hbar) \kappa B_{t-1}^{g,0}$$
$$T_t^2 + (1-\hbar) \left(B_t^{g,0} - (1-\rho) B_{t-1}^{g,0} \right) = G + (1-\hbar) \kappa B_{t-1}^{g,0}.$$

Given taxes, the budget constraint of the savers has now two unknowns: savers' consumption C_t^s and output $\mathbf{N}_t$[12]

$$C_t^{s,0} + T_t^0 + \frac{\chi}{1-\chi} \left(\bar{B}_t^h - R\bar{B}_{t-1}^h \right) + \frac{1}{1-\chi} q_t^0 \left(B_t^{g,0} - (1-\rho) B_{t-1}^{g,0} \right) = \mathbf{N}_t^0 + \frac{\kappa}{1-\chi} B_{t-1}^{g,0} + T_s$$
$$C_t^{s,1} + T_t^1 + \frac{\chi}{1-\chi} \left(\bar{B}_t^h - R\bar{B}_{t-1}^h \right) + \frac{1}{1-\chi} (1-\hbar) \left(B_t^{g,0} - (1-\rho) B_{t-1}^{g,0} \right) = \mathbf{N}_t^1 \left(\frac{1-\Delta-\chi}{1-\chi} \right) + \frac{\kappa}{1-\chi} B_{t-1}^{g,0} + T_s$$
$$C_t^{s,2} + T_t^2 + \frac{\chi}{1-\chi} \left(\bar{B}_t^h - R\bar{B}_{t-1}^h \right) + \frac{1}{1-\chi} (1-\hbar) \left(B_t^{g,0} - (1-\rho) B_{t-1}^{g,0} \right) = \mathbf{N}_t^2 + \frac{\kappa}{1-\chi} B_{t-1}^{g,0} + T_s$$

Fortunately, we have another equation in those two variables, which is the value of the product $C_{s,t} \cdot u_{s,t}^{\frac{\psi-1}{\psi}}$ from the stochastic discount factor calculations. Note that the values of output thus computed depend on the guess of $B_{t-1}^{g,0}$, so this is the moment to iterate over that guess until the tax rate $\frac{T_t^0}{Y_t^0}$ coincides with the pre-specified one. After that, we obtain the consumption

[12] Under wage rigidity, output and the labor supply of savers are linked through the rationing equation $W_s N_{s,t} = \mathbf{W}\mathbf{N}_t$. In the case of default ($\mathcal{H}_t = 1$) the savers bear the TFP cost Δ via profits of the firms (and wage rigidity)

of borrowers as

$$
\begin{aligned}
C_t^{b,0} + T_t^0 &= \mathbf{N}_t^0 + \bar{B}_t^h - R\bar{B}_{t-1}^h + T_b \\
C_t^{b,1} + T_t^1 &= \mathbf{N}_t^1 + \bar{B}_t^h - R\bar{B}_{t-1}^h + T_b \\
C_t^{b,2} + T_t^2 &= \mathbf{N}_t^2 + \bar{B}_t^h - R\bar{B}_{t-1}^h + T_b.
\end{aligned}
$$

Finally, we update the value function V_t of the savers using the recursion (17). This allows us to move into the previous period.

3.4 Simulations

Let us now consider the dynamics of the model. The risk of default is stochastic and given by

$$
\Pr\left(\delta_{t+1} = 1\right) = \epsilon_t \pi\left(B_t^g\right)
$$

where ϵ_t is an exogenous process. We are interested in public deleveraging in an economy that is already depressed. We do so by considering the following sequence of shocks:

- At $t = 0$ the economy is in steady state with $\epsilon_0 = 0$.
- At $t = 1$ private deleveraging starts and last for 5 years, until $t = 20$. Private debt decreases from 80% to 70% of steady state GDP.
- At $t = 5$, there is a shock to sovereign risk: $\epsilon_5 = 1$.
  - We consider a "no risk" benchmark where private deleveraging is the only shock to the economy. We calibrate the initial debt stock and the tax rate so that debt accumulates until reaching 110% of steady state GDP in this benchmark.
  - The government commits to a 10-year deleveraging path, and we consider different starting dates. Sovereign debt decreases in each case to 90% of steady state GDP. We hold fixed the ending date, so delaying is compensated by more aggressive deleveraging. In an extension, we consider the case of pure delaying. In our cali-

bration, precautionary effects are strong enough to make pure delay unambiguously undesirable.

– The deleveraging path involves increasing the tax rate to a level τ_1. We pick this number so that, when our backward algorithm reaches period 5 (when the sovereign risk shock hits), sovereign debt coincides with that under the benchmark.

We summarize these choices in the maximization problem that the government faces in period 5. Given the path of private deleveraging $\left\{\bar{B}_t^p\right\}_{t=5}^{\mathcal{T}}$, as well as the initial steady state, the government chooses a path for government debt $\{B_t^g\}_{t=5}^{\mathcal{T}}$ to maximize

$$\mathcal{W}\left(\{B_t^g\}_{t=5}^{\mathcal{T}}\right) = (1-\chi)\, V_5^s\left(\{C_t^s, N_t^s\}_{t=5}^{\mathcal{T}}\right) + \chi V_5^b\left(\left\{C_t^b, N_t^b\right\}_{t=5}^{\mathcal{T}}\right)$$

where the allocation $\left\{C_t^s, N_t^s, C_t^b, N_t^b\right\}_{t=5}^{\mathcal{T}}$ and the value functions $\left\{V_5^s, V_5^b\right\}$ correspond to the equilibrium that arises when the government chooses the path $\{B_t^g\}_{t=5}^{\mathcal{T}}$. The rest of our assumptions map into restrictions on the path of debt. In reality, the government chooses the starting date of public deleveraging. Before that, tax collections remain constant (as a percent of GDP) as they were in the initial steady state; while after deleveraging has started the tax collections to GDP ratio jumps to a higher number, where it stays until public debt hits the target level at the end of the process. Hitting the debt target at the end of the public deleveraging period pins down τ_1, the ratio of tax collections to GDP during the deleveraging.

Table 2 summarizes the parameters used in our simulations. Values are chosen to follow as closely as possible the possibility of a hypothetical Greek-style default in the Italian economy during the crisis.

Table 3 contains the parameters that describe debts and what happens in default.

Figure 6 shows the path of private $\bar{B}_t^h$ and public debt conditional on no default $B_t^{g,0}$. We consider a model with standard parameters and a risk aversion of 10 to capture significant risk premia. When public deleveraging starts later, debt accumulates initially as savers require a discount on government bonds.

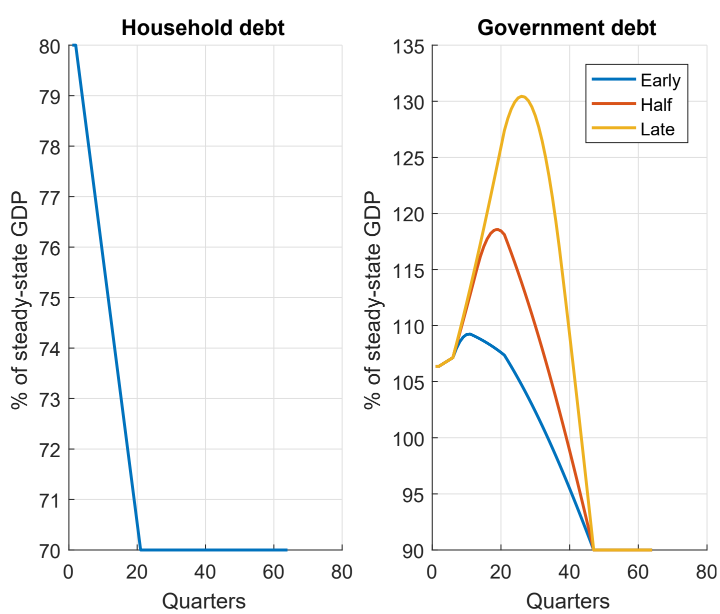
Table 2: Parameter values

Parameter	Description	Value	Target
β	Savers' discount	0.995	2% annual interest
β_b	Borrower's discount	0.972	12% annual interest
χ	Proportion of borrowers	0.5	Standard
φ	Inverse Frisch elasticity	1	Standard
G	Government consumption	$20\% \times \bar{Y}$	Italy 1999-2008.
$\mathbf{C}$	Steady-state consumption	1	Normalization
ψ	Inter-temporal substitution	1	Standard
γ	Coefficient of risk aversion	10	Standard for asset pricing moments
α	Consumption share in utility function	0.40	$W = 1$ (normalization)
κ_n	Labor endowment	2.78	Work week of 36 hours

Table 3: Debt and deleveraging parameters

Parameter	Description	Value	Target
ρ	Persistence of public debt	5%	Duration of Italian debt 2010
$\hbar$	Haircut	50%	Greek default
Δ	Deadweight loss after default	10%	Greek default
θ	Hazard of low productivity state	1/10 per year	Greek default
B_T^g	Final public debt	$4 \cdot \bar{Y} \cdot 90\%$	Italy in Great Recession
B_0^h	Initial private debt	$4 \cdot \bar{Y} \cdot 80\%$	Italy in Great Recession
B_T^h	Final private debt	$4 \cdot \bar{Y} \cdot 70\%$	Italy in Great Recession
T_h	Length of private deleveraging	20	Italy in Great Recession

Figure 6: Deleveraging Paths for Private and Public Debts

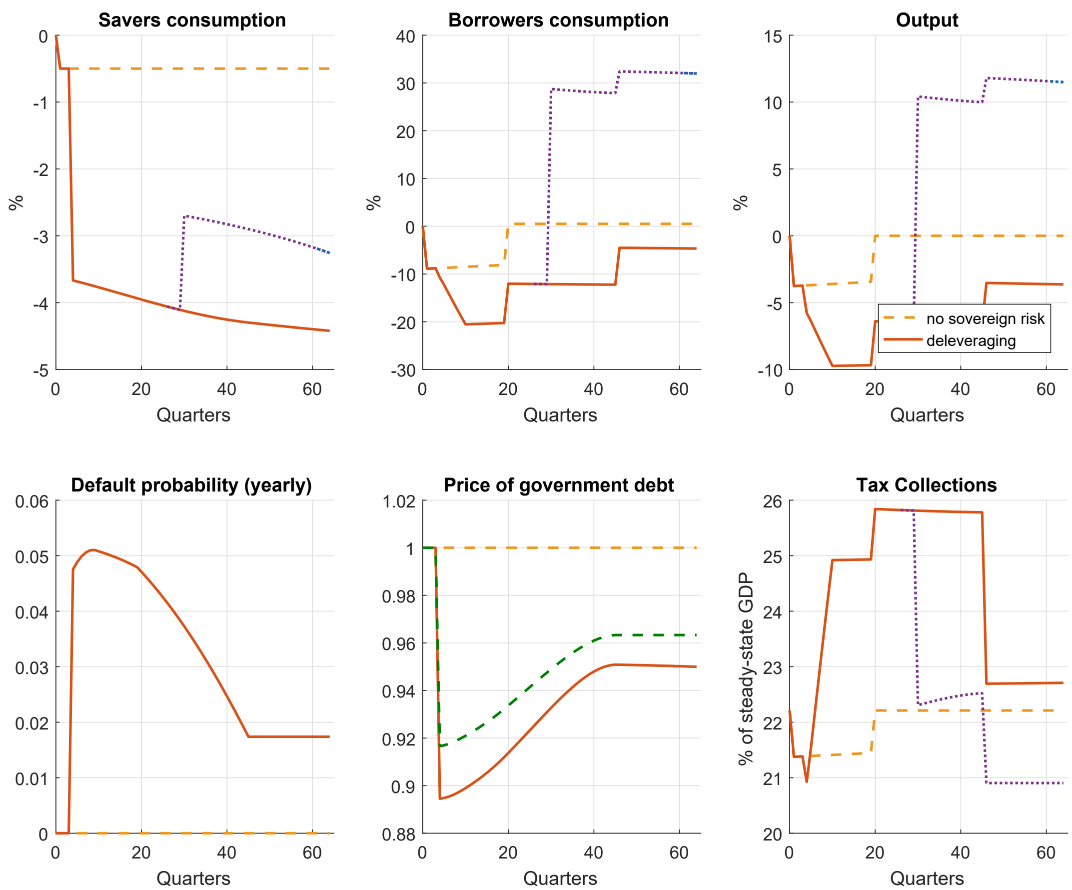


Note: Model with π^{crisis}.

Figure 7 plots the paths of real outcomes, starting with savers' consumption. Private deleveraging implies smaller interest payments from borrowers to savers in the long run. Since our savers are permanent income agents they lower their consumption. With $r = 2\%$ and a decrease of 0.1 GDP this predicts roughly a 20 basis points drop in consumption, just from the long run effect. In addition, there is the capitalized value of lost output because of the recession, which is of the same order of magnitude. So absent all other shocks, savers' consumption drops by around 0.5%.

Then, at $t = 5$, we switch on the credit risk. The price of government debt drops by roughly 10%. Savers make a capital loss and their consumption drops further. Then much depends on what the government does.

Figure 7: Sample Paths with Early Sovereign Deleveraging



Notes: Each period is one quarter, $\gamma = 10$. Private deleveraging occurs in all cases, from period 1 to period 20, and reduces household debt from 0.8 GDP to 0.7 GDP. Yellow dashed line has no sovereign risk or deleveraging. Solid red line has sovereign risk and deleveraging, and plots the paths conditional on no default. Dotted purple line is one path where sovereign default occurs in period 30. Dashed green line corresponds to the risk-neutral price of government debt, in the deleveraging case.

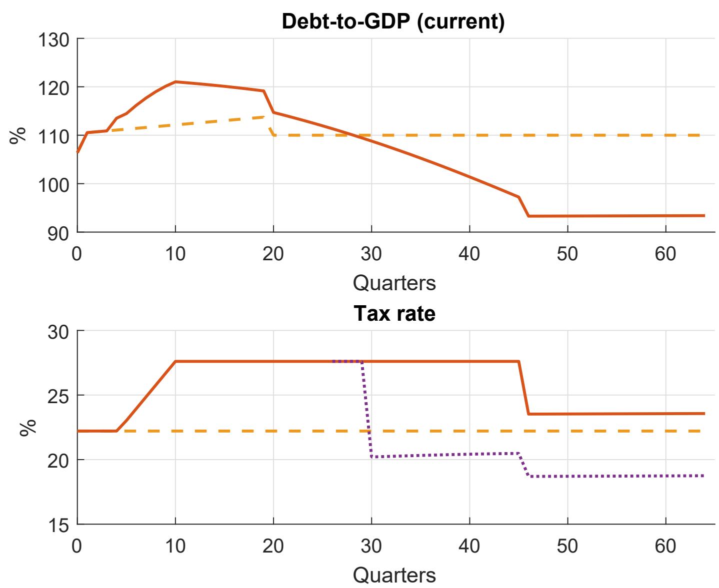
If the government reduces leverage right away as in Figure 7, savers' utility appreciates as time passes without default.[13] In this case, the pricing of sovereign debt is interesting. The green dashed line is the price of debt if it were priced by risk neutral investors, so it captures exactly the expected loss. The gap with the red line is the risk premium because our savers are risk averse. The risk premium is roughly constant and even *grows* very slightly once debt settles at a safe level. This paradox is explained by the consumption path of savers. When debt is unsafe, they choose consumption almost *as if* default would happen for sure. This

[13]The path of consumption is a bit counterintuitive here, because of the non-separability of consumption and leisure. With separable preferences, consumption would trend up as time passes without default.

behavior subsides as debt becomes safer, which makes the potential loss in case of actual default worse.

Borrowers' consumption follows the path of deleveraging, and output net of taxes. They are hit by two negative shocks: first, their own deleveraging shock; second, the drop in savers' consumption at time 5. In most cases, discrete jumps in taxes would induce large drops in their consumption. Borrowers would then want to save by choosing lower levels of debt than are allowed for them. We dodge this complication by giving the government a smoother path for the tax rate.

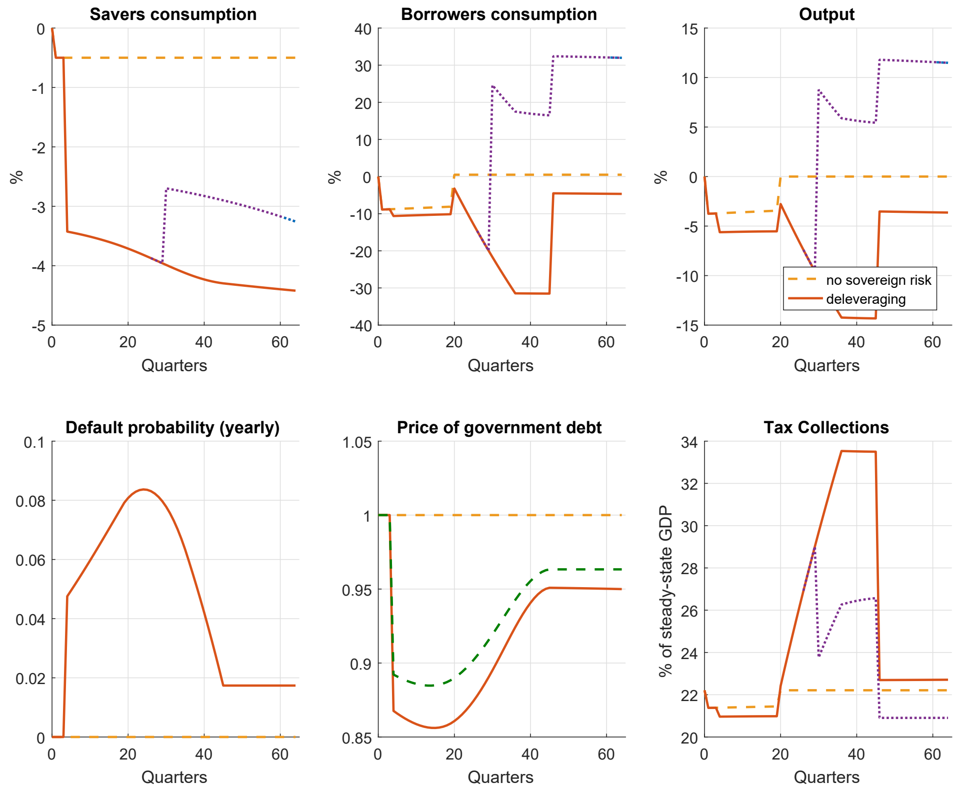
Figure 8: Sample Paths with Early Sovereign Deleveraging



Notes: Each period is one quarter, $\gamma = 10$. Private deleveraging occurs in all cases, from period 1 to period 20, and reduces household debt from 0.8 GDP to 0.7 GDP. Yellow dashed line has no sovereign risk or deleveraging. Solid red line has sovereign risk and deleveraging, and plots the paths conditional on no default. Dotted purple line is one path where sovereign default occurs in period 30.

Figure 8 shows the dynamics of the debt-to-GDP ratio and tax rates. The tax rate increases smoothly between periods 5 and 10 in the simulation with sovereign deleveraging, while output and consumption of borrowers fall. This plunge in output contributes to the initial increase of the debt-to-GDP ratio.

Figure 9: Sample Paths with Late Sovereign Deleveraging



Notes: Each period is one quarter, $\gamma = 10$. Private deleveraging occurs in all cases, from period 1 to period 20, and reduces household debt from 0.8 GDP to 0.6 GDP. Yellow dashed line has no sovereign risk or deleveraging.Solid red line has sovereign risk and deleveraging, and plots the paths conditional on no default. The dotted purple line is one path where sovereign default occurs in period 30. The dashed green line corresponds to the risk-neutral price of government debt, in the deleveraging case.

Figure 9 shows the case where the government waits for private deleveraging to end before starting its own deleveraging. There is less austerity in the short run, but savers have to live with high credit risk for 5 years, which depresses their consumption. Figure 10 compares the welfare losses as a function of the delay before deleveraging, compared to the case with only private deleveraging and no sovereign risk. The right panel shows that the output loss is roughly constant but that it is minimized when the government about two years before starting its own deleveraging.

Proposition 3. *In the model calibrated to the eurozone crisis, we find that:*

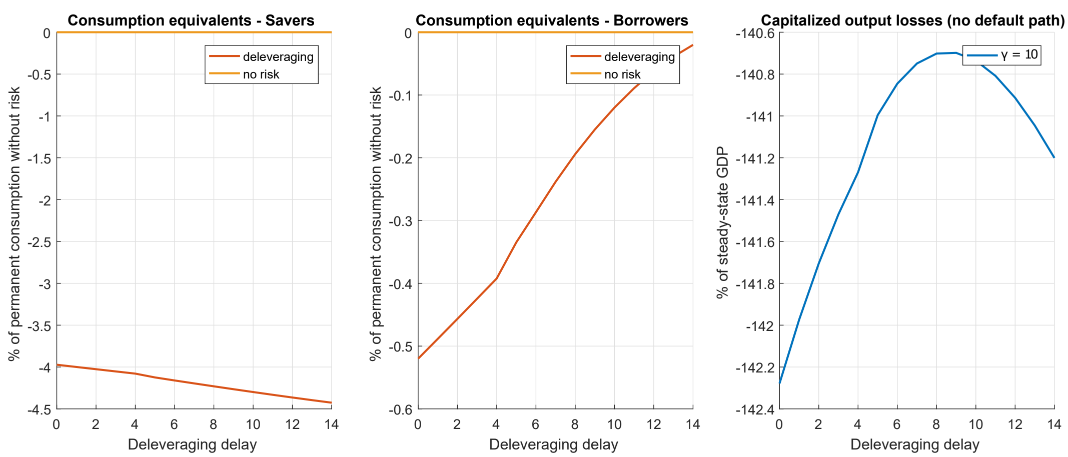
(i) there is disagreement between savers and borrowers: savers prefer an early deleveraging process, while borrowers prefer a late deleveraging process;

(ii) in crisis times (high π) there is an interior solution for when to start public deleveraging, while in normal times it is optimal to delay.

(iii) risk aversion has a large impact on the output loss and on welfare when sovereign debt is risky;

Figure 10 illustrates the disagreement between borrowers and savers in our baseline calibration. Savers unambiguously prefer the deleveraging delay to be short, reflecting their exposure to default risk. Borrowers, on the other hand, prefer deleveraging to be postponed, both because they care more about the current value of output and would rather prevent making the recession worse and because they understand that a default implies lower taxes in the future. The third panel shows that capitalized output losses (over the path conditional on no default) are minimized when the deleveraging process starts 2 years after the crisis started. This is not a welfare conclusion but it illustrates the point that austerity (as measured by delay) has ambiguous effects on output over time, even only considering plans that are successful and not interrupted by a default.

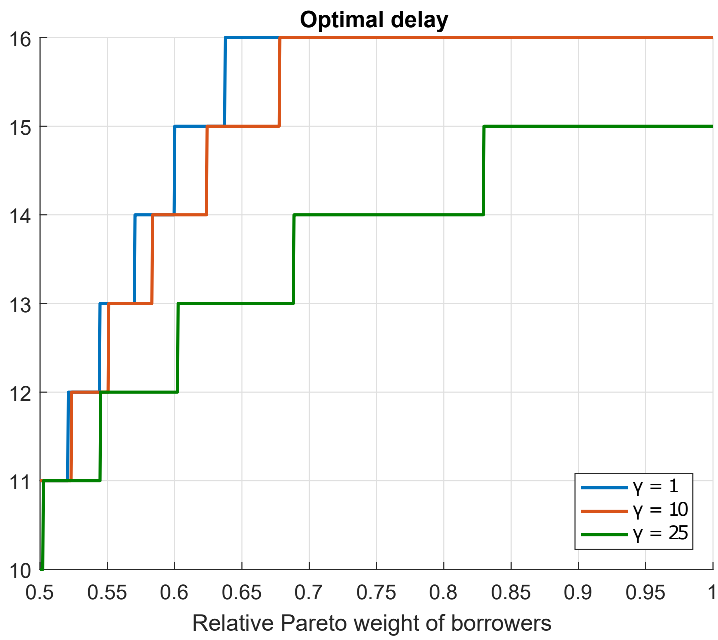
Figure 10: Welfare and Deleveraging Delay in a Crisis



Notes: Horizontal axis is the delay in quarter between the risk shock and the start of sovereign deleveraging. Model with π^{crisis} and $\gamma = 10$. Vertical axis measures welfare in consumption equivalent units. Output losses are capitalized with the borrower's discount factor.

Figure 11 below shows which deleveraging plan a planner would pick, as a function of the relative preference for borrowers in its social welfare function. In our calibration, a utilitarian planner would wait for two and a half years before starting the deleveraging process. We call this an interior solution because the planner neither wants to start right away nor delays forever. Rather, the optimal policy balances fast accumulation of debt in the 'delaying' phase against decoupling the tax increase from the original recession.

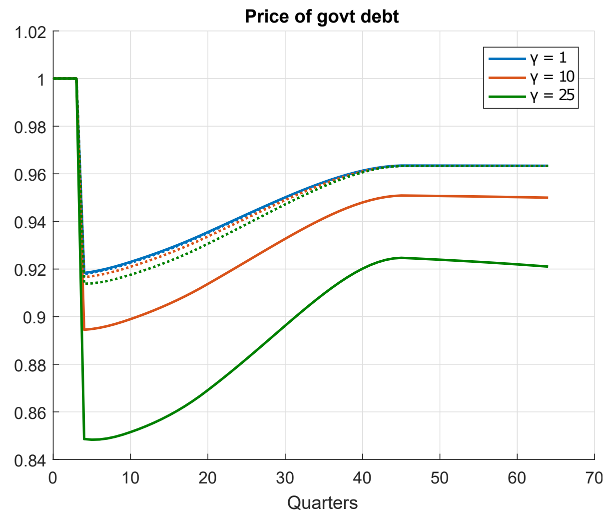
Figure 11: Optimal deleveraging delay and risk aversion



Notes: Model with π^{crisis}. Vertical axis measures the optimal deleveraging delay in quarters.

Figure 13 compares the output losses on the no-default path in crisis times and in normal times, for different levels of risk aversion. Risk aversion significantly affects output. This stands in stark contrast with the results in Tallarini (2000). In our model, γ has a large impact on risk premia in equilibrium but also on the macro quantities. This is because, as savers become more risk averse, they plan their consumption path so that it would not drop much in case of default. Risk averse savers cut their consumption more when credit risk increases, so that the actual default outcome becomes relatively less severe for them. This diminishes the direct impact of risk aversion on sovereign spreads. In our model, an increase in risk aversion has a sizable impact on macroeconomic dynamics, and a relatively weaker impact on asset prices. Figure 12 illustrates this finding.

Figure 12: Debt Prices and Risk Aversion

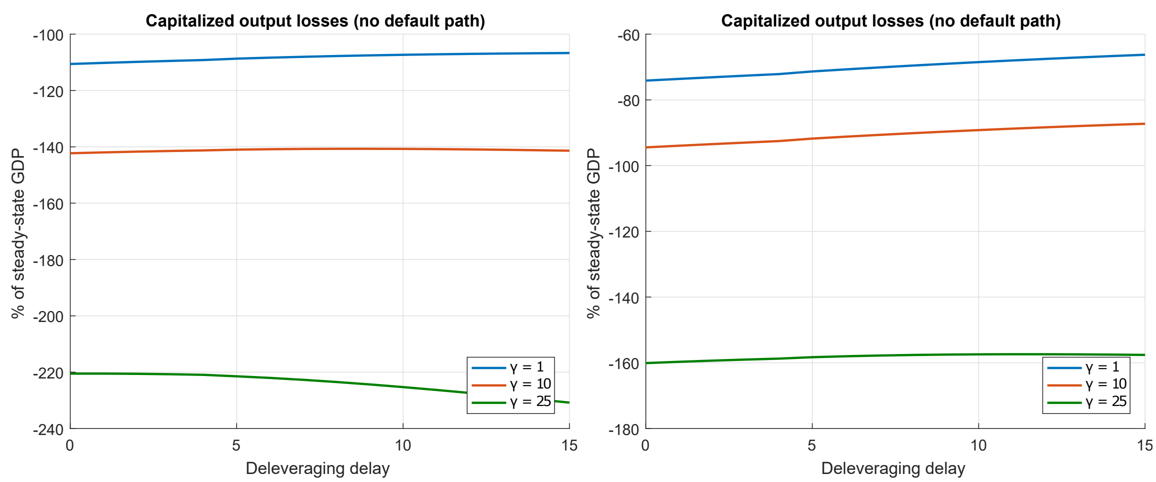


Notes: Model with π^{crisis} and early sovereign deleveraging. Dotted lines show the risk-neutral price of debt in each case.

Figure 13 also shows that, in crisis times, delaying yields lower present value of output. In normal times and for reasonable risk aversion, on the other hand, delaying increases the NPV of output.

Figure 13: Output loss and Deleveraging Delay in Normal Times vs Crisis Times

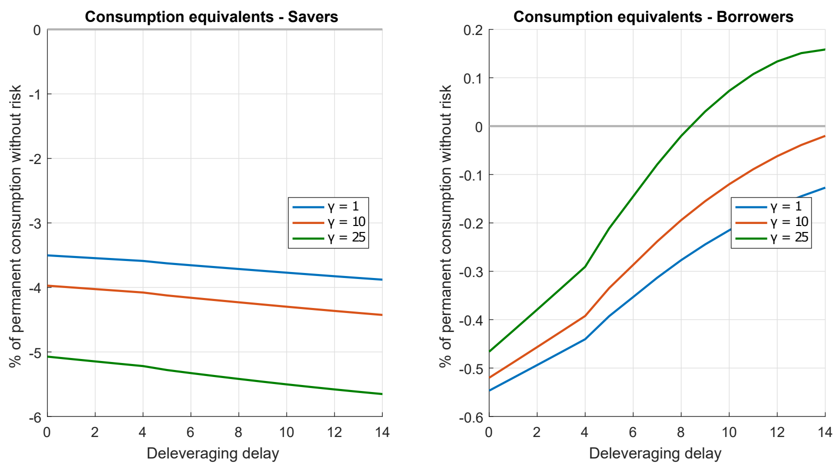
(a) Crisis Times (b) Normal Times



Notes: Output losses are capitalized with the borrower's discount factor.

Figure 14 compares the welfare losses in consumption equivalent units for different delays and different values of the risk aversion parameter. Increasing γ has a large impact on welfare for savers, and a modest one for borrowers. The savers' reaction to the sovereign risk makes borrowers not want to delay indefinitely. For a somewhat high level of risk aversion ($\gamma = 25$), this effect can be seen clearly in the figure, which implies a preferred delay of about 3 years for borrowers.

Figure 14: Welfare and Deleveraging Delay for Different Risk Aversions



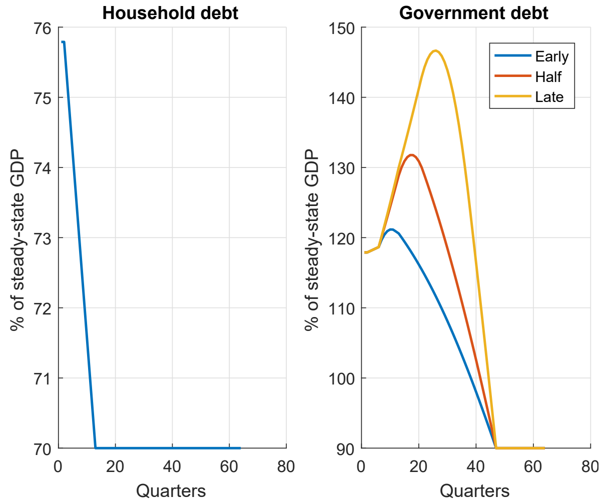
Notes: Horizontal axis is the delay in quarter between the risk shock and the start of sovereign deleveraging. Model with π^{crisis}. Vertical axis measures welfare in consumption equivalent units.

The general point here is that there is more agreement about the speed of deleveraging during a financial crisis, when risk aversion and spreads are high. Disagreement increases when the crisis recedes, which seems consistent with casual evidence from the post-Great Recession era.

3.5 Time Consistency

Up to here we have discussed how a government would choose to go about its deleveraging subject to the exogenous default probability but nonethteless with complete commitment to actions conditional on no default. We believe that to be the relevant assumption: fiscal plans to quickly reduce sovereign debt often come from agreements with creditors and international institutions. Breaking such plans entails renegotiations from haircuts are not out of the question.

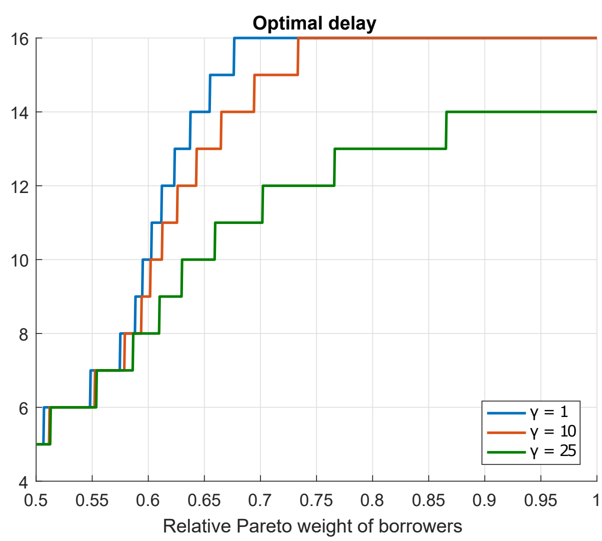
Figure 15: Deleveraging Paths for High Initial Public Debt



Note: Model with π^{crisis} and high initial debt

While characterizing the optimal policy under discretion is beyond the scope of this paper, we offer some insight into the strength of the temptations to not follow through with the plan. We do this by starting the economy at the point at which it is supposed to start deleveraging under the optimal policy. Figure 15 shows the paths of private and public debts. Private deleveraging is half of the way, while public debt has accumulated.

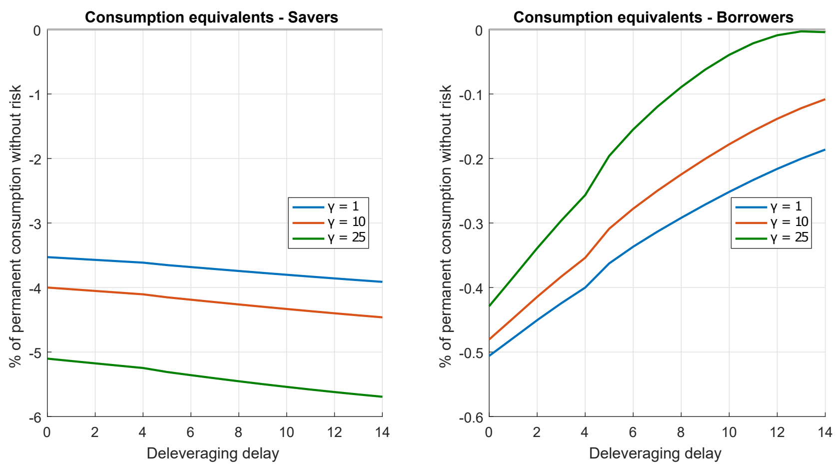
Figure 16: Optimal Policy for High Initial Public Debt



Notes: Model with π^{crisis} and high initial debt. Vertical axis measures the optimal deleveraging delay in quarters.

The optimal delays in this high-debt environment are shown in Figure 16. Even though this policy does not coincide with what the original policy's prescription, the optimal delay is reduced significantly. Disagreement between borrowers and savers does not subside, as can be seen in Figure 17. Rather, borrowers favor (yet) more delay because they prefer taxes to increase later rather than sooner. We conclude that it may be a good idea to attach costs to breaking the deleveraging plan.

Figure 17: Welfare and Delay for Different Risk Aversions



Notes: Horizontal axis is the delay in quarter between the risk shock and the start of sovereign deleveraging. Model with π^{crisis} and high initial debt. Vertical axis measures welfare in consumption equivalent units.

4 Conclusion

We analyze the tradeoff between sovereign risk and fiscal austerity in an economy with heterogenous agents where domestic savers hold (most of) the government debt. The negative impact of fiscal austerity on growth is muted by the endogenous response of savers.

We find that borrowers and savers tend to disagree about the optimal path of sovereign deleveraging, even though they are equally exposed to the recessionary impact of fiscal austerity. Savers prefer early deleveraging because it lowers the risk of default. Borrowers are more worried about aggregate demand and prefer the bulk of sovereign deleveraging to happen after private deleveraging is over. This might explain why it is so difficult to find political consensus regarding fiscal policy in the aftermath of a financial crisis.

From a macroeconomic perspective, we find that the sovereign risk channel creates a sizable drag on aggregate demand. The extent of the drag depends on the risk aversion of the savers. The more risk averse the savers are, the more they cut spending when the risk shock hits. Because they cut more initially, the drop in case of actual default is smaller, and this limits the rise in the risk premium. Thus, unlike in Tallarini (2000), we find that risk aversion has a significant impact on quantities, and not only (or even mainly) on prices.

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