Distributional Effects in Sovereign Debt Policy*

Juan M. Morelli† NYU Stern

Francisco Roldán‡ NYU

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PRELIMINARY AND INCOMPLETE

Abstract

We study a closed economy in which heterogeneous households use sovereign debt as the vehicle for savings. A benevolent but discretionary government uses its public debt policy to influence the distribution of consumption through households' wealth. We perform default and taxation experiments that highlight the forces at play and their interaction. We find that the social value of public debt as a savings vehicle allows the government to sustain borrowing in the absence of explicit penalties, while at times the distribution of wealth may create an incentive for sovereign default.

INTRODUCTION

Sovereign debt has at least two interactive features with differential effects across the distribution of households' wealth. Firstly, debt policy gives a government the ability to smooth taxes over time, with its consequent implicit transfers of resources. Debt issuance allows for a reduction in current taxes, defining an implicit transfer from bond buyers to tax payers. The reverse transfers occur in the future when the government raises taxes to repay its debt. In addition, sovereign default takes resources from bondholders to taxpayers. A second feature of public debt is that agents can use their debt holdings as a savings vehicle for self-insurance in an economy with income shocks and borrowing constraints. These effects interact in the sense that debt issuance and default may ease the constraint and increase consumption of a low-wealth household. It is thus of special interest for a benevolent government to understand these interactive features, and the feedback between the former and the distribution of income and wealth.

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†e-mail: *jml934@nyu.edu*

‡e-mail: *froldan@nyu.edu*

In our model, a benevolent government chooses its debt policy to optimize the welfare of a continuum of heterogeneous agents in an economy subject to aggregate fluctuations. In order to implement the first-best, the government would need access to person-specific lump-sum taxes. We assume that those are not available. Rather, we assume that the progressivity of the tax scheme is fixed, and that the government can collect lump-sum taxes and issue non-contingent defaultable debt. In this economy, it is possible to shape the distribution of consumption by actively affecting the distribution of wealth. However, decisions about government debt can only directly impact the distribution of bondholdings. In reality, those distributions are related but not identical. The question then becomes how related they are. In addition to direct and indirect holdings of public debt, sovereign credit risk has spillovers over the private sector's costs of funding, and actual defaults are associated with recessions.

Our paper consists of an empirical and a theoretical section. Our first contribution is the identification of the direct and indirect effects of public debt policy on households portfolios, as well as an evaluation of the economic relevance of exposures of private agents to sovereign risk. On the theoretical side, we develop a model of debt policy in which the government reacts in real time to the current distribution of wealth. We focus on a closed economy for two reasons. First, it is better suited for countries in the peripheral eurozone. The majority of the debt is held by agents within the eurozone. Secondly, we want to show that debt is sustainable without explicit penalties, and that default may still occur on domestic agents. On our quantitative section we find a rich feedback between the shape of the distribution and the government policies. The optimal level of debt trades off the benefits of better insurance against a higher level of inequality that is required to sustain higher aggregate levels of bondholdings. As inequality gets too high, default incentives emerge.

Discussion of the Literature This paper relates to several strands of literature. We depart from the canonical models that build on Eaton and Gersovitz (1982) and Arellano (2008) by considering the impact of the distribution of households on default incentives and costs. Moreover, in those models debt is priced by risk-neutral foreigners, while in ours it is held internally, which means that both the marginal buyers and their stochastic discount factor are endogenous.

Other papers have emphasized internal costs of sovereign default. Mendoza and Yue (2012) assume that domestic firms lose access to some imported inputs after a default, which reduces their productivity. Others such as Bocola (2015), Gennaioli et al. (2014), and Perez (2016) focus on the disruption of financial intermediation in the domestic market. We regard these papers as complementary to ours.

We are certainly not the first to introduce distributional considerations in sovereign debt policy. The seminal contributions of Woodford (1990) and Aiyagari and McGrattan (1998) have thought about the role of public debt as providing liquidity in economies in which incomplete markets curtail savings. We build on these papers by allowing for the distribution of households to evolve over time, which then allows us to think about how this evolution interacts with issuance and repayment incentives going forward. Ferriere (2016) highlights the progressivity of the tax schedule as a key predictor of external default incentives. Romei (2015) studies the distributional effects of different speeds of fiscal consolidation. Most related to us is the work of D'Erasmo and Mendoza (2016), who consider a similar environment but where the government weighs the welfare of different agents according to some predetermined bias for or against creditors, which makes them abstract from the current distribution as a predictor of default incentives. Finally, Guembel and Sussman (2009) and Andreasen et al. (2011), among others, have asked whether the median voter would oppose fiscal plans to repay sovereign debt.

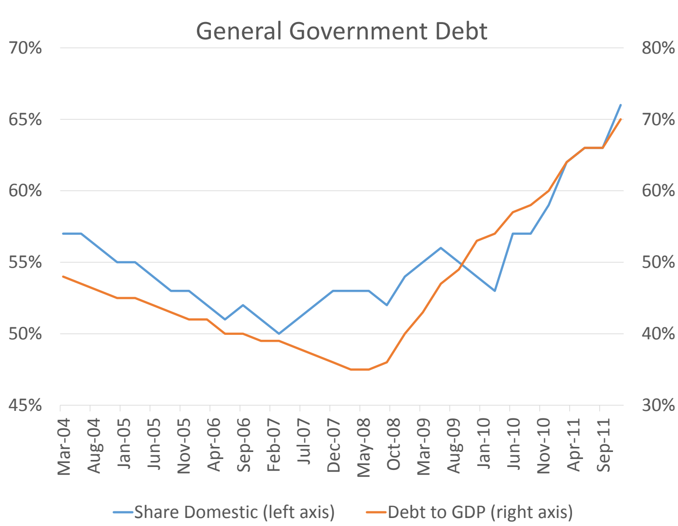
We base our quantitative solution method on Reiter (2010), Algan et al. (2008) and Winberry (2016). We solve for the model with aggregate shocks while avoiding a simulation plus update iterative step. Our algorithm has the advantage that it imposes consistent beliefs between the government and the private sector, thus making sure that the government's policy does not rely on exploiting private agents' forecast errors (Lucas, 1976).

Our baseline model is a closed economy. This allows us to characterize the distributional incentives and costs of sovereign default, while abstracting from external debt. We take the stand that in reality default decisions trade off a transfer from the rest of the world (via forced debt relief) against internal costs such as the disruption of domestic liquidity, and therefore set out to study such internal costs in isolation.

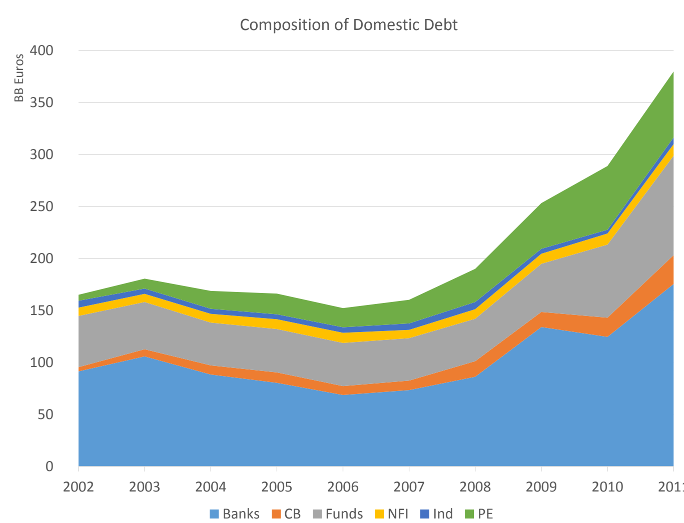
The remainder of the paper is organized as follows. Section 2 presents our empirical analysis. Section 3 lays out our model and defines the equilibrium of the game played by the government and the agents, while Section 4 describes our solution method. Section 5 discusses our preliminary results. Finally, Section 6 (TBW) concludes.

2. Empirical Evidence

As a motivating example, we take the Spanish economy in the years neighboring the European financial crisis. We take data from the Bank of Spain, the Spanish consumers' finance survey (EFF) and the Spanish Treasury. Figure (1a) shows the general government's debt-to-GDP ratio and the share of domestic debt to total debt. The main takeaways are that (i) there is a sharp and sustained increase in the debt-to-GDP ratio after the crisis and (ii) domestic investors are the main holders, in particular during the crisis. Figure (1b) details domestic debt by holding sector. We observe that banks and funds are the main holders.



(A) Sovereign Debt



(B) Composition of Domestic Debt by Sector

As can be seen in Figure (2), the upcoming of the financial crisis was followed by a stark and sustained increase in sovereign risk. This is explained by a strong deterioration in fundamentals: as output and revenues plummeted, the fiscal deficit escalated and so did debt. The government would have little room for market intervention, in particular the rescue of sizable Spanish banks.

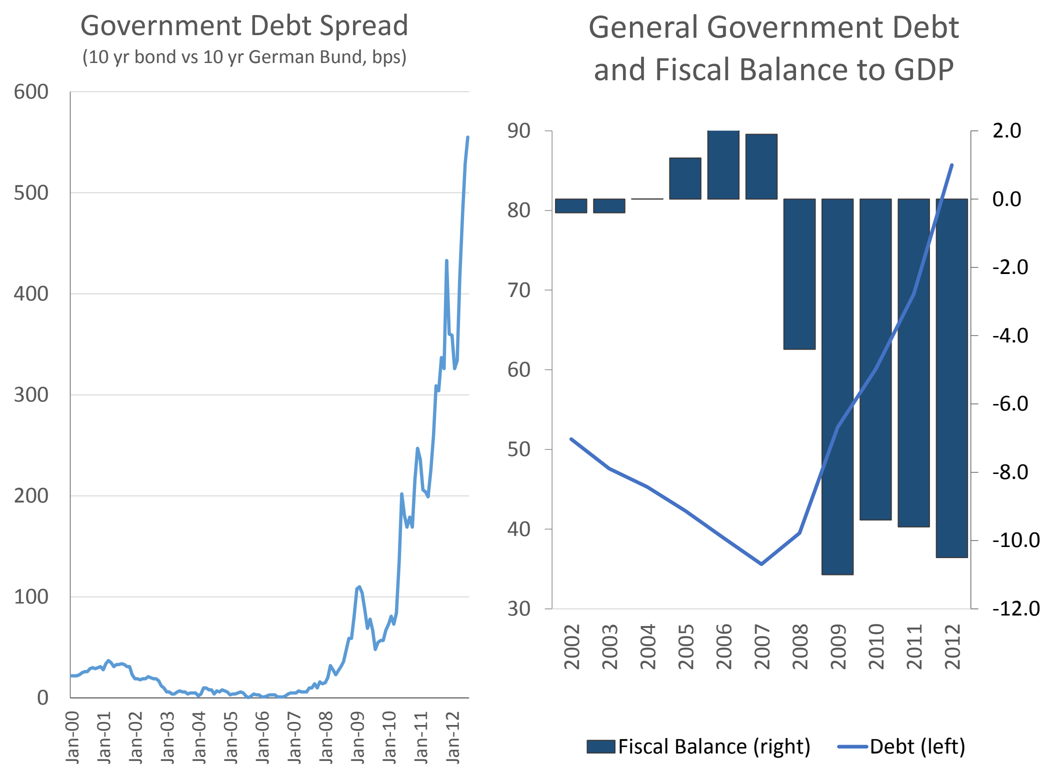


FIGURE 2: SPAIN'S SOVEREIGN RISK

We are interested in characterizing how the issuance of debt was absorbed by Spanish households across the wealth and income distribution. Households, however, tend not to hold government debt directly. They do hold different assets which are themselves exposed to public debt, as we will see. Figure 3 shows the composition of financial assets by percentile group of net wealth. The EEF allows us to distinguish between liquid and illiquid deposits, stocks, pension funds, insurance, investment funds, fixed income, and other financial assets. The portfolios are heterogeneous across the wealth distribution as well as over time. Broadly speaking, stocks represent a high proportion of the portfolio of the very rich, even more so in 2011 at the height of the crisis. Second, direct holdings of government debt, which would fall under the fixed income category, cannot account for more than a few percentage points of households' net wealth in any percentile group or year.

FIGURE 3: PORTFOLIO OF FINANCIAL ASSETS

Pension funds, insurance companies, mutual funds, and banks all hold government debt as assets in their balance sheets. This makes these financial institutions exposed to losses when the spread on government debt increases, and also in case of an actual government default. How much losses would be passed on to the actual households depends on how much debt these institutions hold and also on the type of institution and the type of contract it has with its depositors.

We take the stand that pension funds, insurance companies, and mutual funds will pass through their losses perfectly to the households. That is, we assume that when a household owns a dollar in one of these funds, it indirectly holds one dollar's worth of the portfolio of assets that the fund owns. Table 1 shows the amount of debt that each type of institution holds on average as a proportion of assets in each year.

Year	Insurance	Pensions	Mutual Funds
2002	21%	20%	21%
2005	17%	10%	7%
2008	13%	13%	8%
2011	34%	30%	24%

TABLE 1: FUNDS EXPOSURE TO PUBLIC DOMESTIC DEBT

Using the proportion of assets from Table 1, we can split the households' holdings of pension funds, insurance, and mutual funds between indirect bondholdings, which we can add to the direct holdings, and other assets, which results in Figure 4.

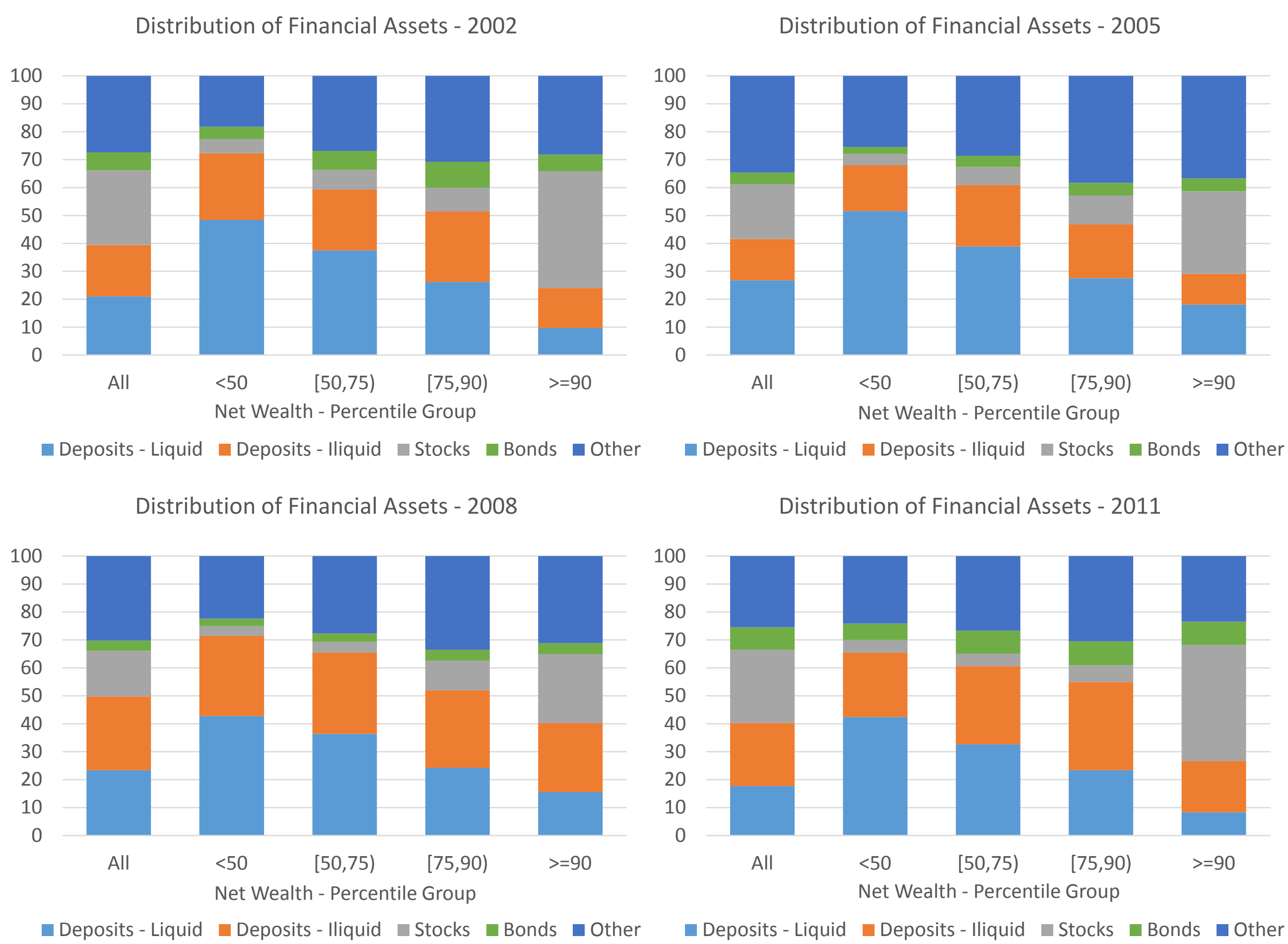


FIGURE 4: EXPOSURE TO DOMESTIC PUBLIC DEBT THROUGH NONBANK ASSETS

The effect is an increase in the average exposure from about 1% or 2% of total financial assets to around 6% to 7%. This exposure is generally higher on the percentiles with highest net wealth, reaching levels of almost 13%. It is also higher for all groups during the crisis, getting values of about 9% in 2011. Figure 5 shows the ratio of this adjusted measure of bond holdings to annual total labor income, by percentile group of labor income. We observe that weights increased on the aftermath of the European crisis, especially for the highest percentile groups.

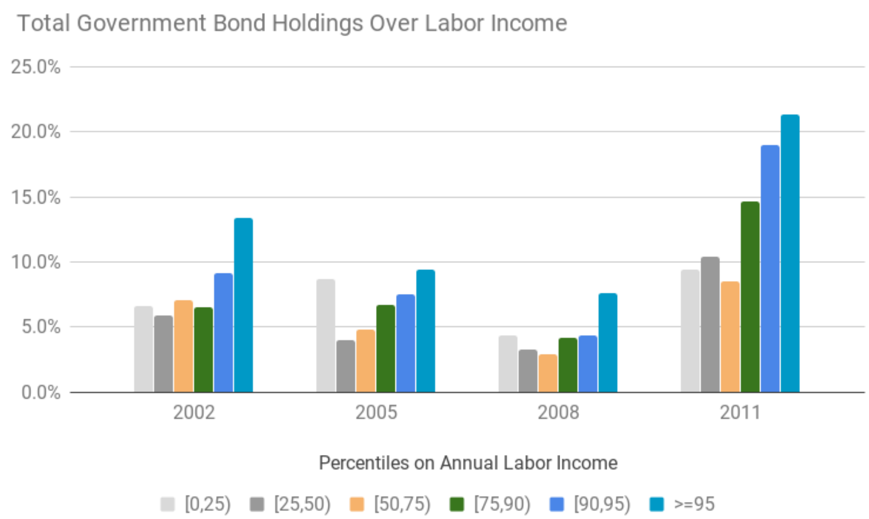


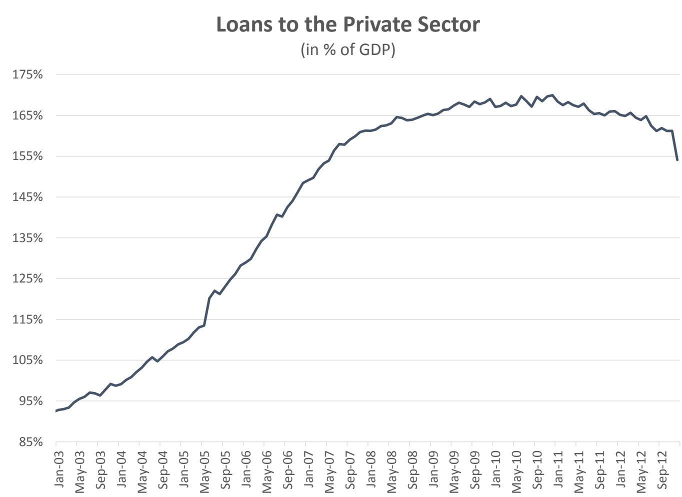
FIGURE 5: DEBT HOLDING TO LABOR INCOME

Finally, we consider the exposure to government debt through bank deposits. These account for above 40% of total financial assets, reaching levels of up to 70% for households with the lowest wealth. Deposits are about riskless in normal times, but the banking sector in Spain was in the midst of a banking crisis. Laeven and Valencia (2012) define a systemic banking crisis as episodes in which (i) there is significant signs of financial distress (bank runs, high non-performing loans and/or bank liquidations); and (ii) there is as significant government intervention in the banking sector (e.g. liquidity support, bank nationalizations, asset purchases, deposit freezes, etc.)[1]. According to their measure, Spain reached a systemic baking crisis exactly in 2011.

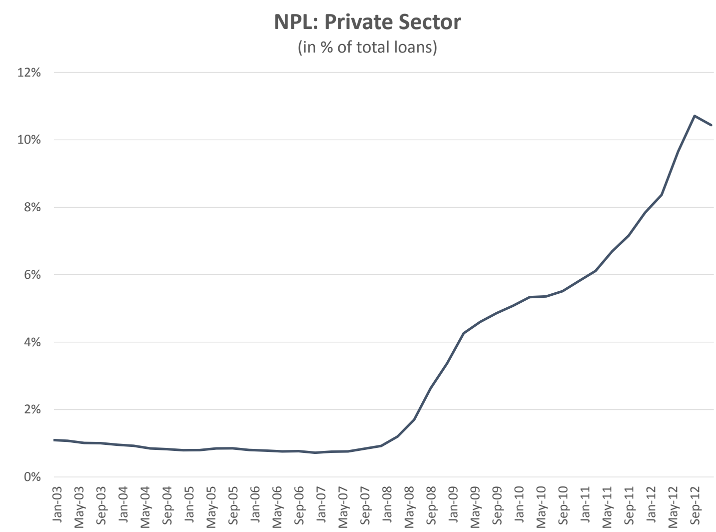
In addition, Balteanu and Erce (2017) have shown that episodes of just a banking crisis on emerging markets have[2], on average, a different behaviour than episodes where a banking crisis is followed by a sovereign debt crisis. In both cases, there is a boom in loans to the private sector, and the ratio of non-performing loans escalates towards the crisis. With the caveat that Spain is not considered an emerging economy, it still is exactly what happened to the country up to 2012, as seen on the first two panels of figure 6 below. As a counterpart, the return on assets and return on equity of Spanish banks was significantly reduced towards 2011. A fragile banking sector leaves the government no other option but to intervene via Central Bank's liquidity provision, and Central Government's asset purchases and capital injections on troubled banks. This implies a boost in public spending that, jointly with the shortfall in revenues created by the downturn in economic activity, significantly deteriorates fiscal balances[3]. In particular, in episodes where the deterioration is strong enough, and public debt suffers a sharp increase, the banking crisis is inevitably followed by a public debt crisis[4]. On this respect, figure

[1]Refer to paper paper for further details. They have an updated version of their online database.
[2]Their sample contains 104 emerging markets and developing countries, covering years from 1975 to 2007.
[3]See appendix for details on measured taken by the Spanish government during the bank crisis
[4]Out of the 81 systemic banking crisis analyzed by Balteanu and Erce (2017), 18 of them ended up with a debt crisis.

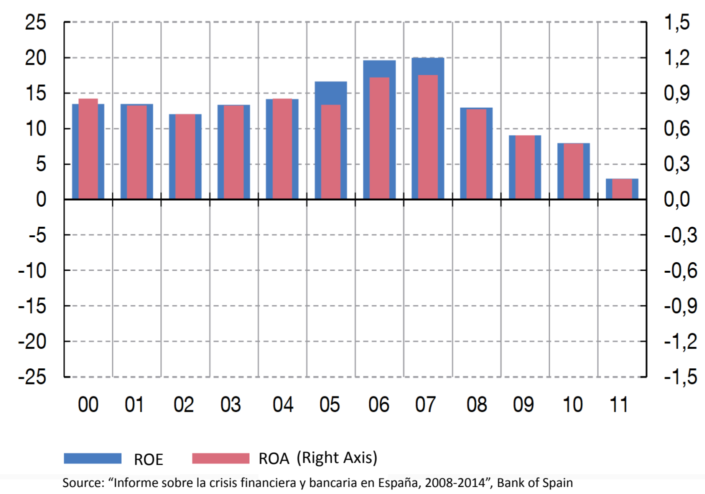
7a shows that Spain was no different. Of course, default on public debt would imply a significant second blow to the banking system. Such a risk was actually priced by markets. On figure 7b below we observe a close link between sovereign and banks' default risk[5].



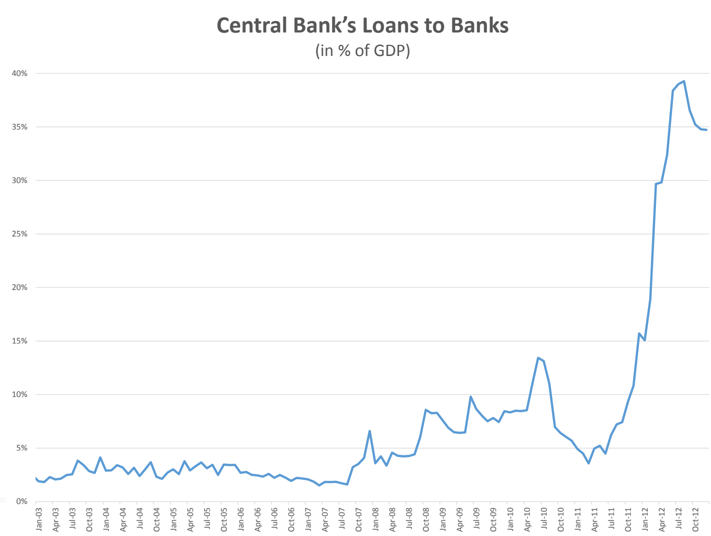
(A) Credit Boom



(B) Stark Rise in Deliquency



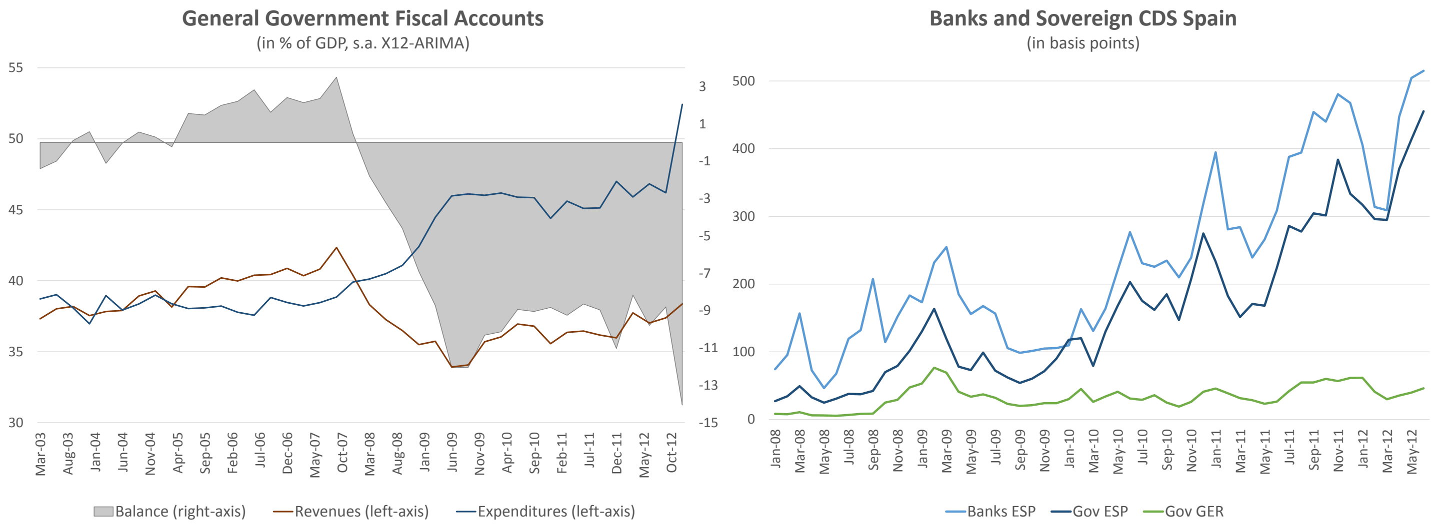
(C) Fall in Returns



(D) Liquidity Provision During Bank Crisis

Figure 6: Overheating of the Banking Sector

[5]These CDS are for the major Spanish banks. They were in good financial health prior to the banking crisis and adquired many of the troubled banks from 2010 through 2014. Nonetheless their default risk increased given the downturn in economic activity, stark fall in returns on loans, and rise in sovereign risk



(A) FISCAL DETERIORATION

(B) SOVEREIGN AND BANKS CREDIT DEFAULT SWAP

FIGURE 7: FISCAL ACCOUNTS AND DEFAULT RISK

Overall, Spain had a banking crisis that almost turned into a twin bank-debt type of crisis. There are several reasons why a looming Greek-style crisis did not materialize. Being a sizable eurozone member, the political costs of such a policy would have been high, not to mention the economic sanctions that peer countries could have imposed. Furthermore, to the extent that default is not selective, given the tight network of the european banking system, one would expect immediate spillovers to other eurozone countries. This, in turn, would have had a negative feedback on the Spanish economy. Regardless of the specific reasons why the government decided not to default on its debt, we are interested in analyzing what would have been the distributional effects should default had ocurred.

The next step is to measure the expected haircut on deposits in case the government defaults. We use data from Philippon and Salord (2017) who study European banks resolution in the European crisis. They focus on the case of Cyprus, where banks owned by the main Euro Area members[6] suffered important losses due to the European crisis and had incur bail-ins and bail-outs.

The authors estimate that the average total recapitalization need was around 17.4% of pre-crisis assets. On average, private investors (shareholders, junior debt holders and some senior debt holders) provided about 33% of that need via losses in equity (91%), junior debt (53%) and senior debt (14%). The other 2/3 came from government intervention. However, in contrast to the scenario we consider for Spain, there was no sovereign debt crisis in Cyprus. We are interested in the case where the Spanish government will not be able to provide for the remaining need of recap funds, and this will have to be obtained from senior debt holders and deposits.[7] We consider different scenarios as to how the losses are distributed between senior debt and deposits. Table 2 shows what would be the average loss[8] in deposits under these different assumptions:

[6] The banks were Dexia 1, Amagerbanken, Anglo-Irish, Hypo Real Estate, Dexia 2, Bankia, SNS Reaal Group, Laiki and Alpha Bank, owned by other main countries from the Euro Area.

[7] Further comparisons will be made to the case of countries that went through a banking and debt crisis such as Argentina and Uruguay in 2002.

[8] The average was computed using times series for total assets and deposits, potential losses are fairly stable across time.

Scenario	SD Loss	Dep. Loss
Extreme	25%	14%
Mild	50%	10%
Conservative	75%	5%

TABLE 2: EXPECTED LOSSES ON DEPOSITS

Figure 8 assumes a 50% haircut on public debt that triggers a bank crisis, with losses for depositors of 10%, which fall on our 'mild' scenario. Overall, the public and banking crisis would induce a fall of between 8% and 10% of financial assets.

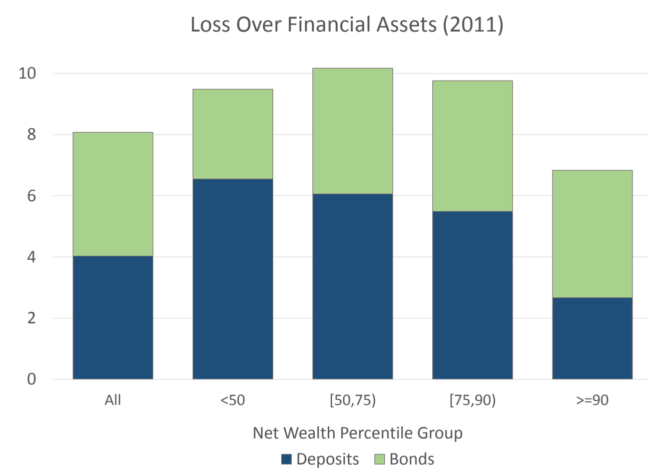


FIGURE 8: EXPECTED LOSS UPON 50% HAIRCUT

We showed that even though debt is mainly concetrated on the wealthiest households, it does represent a relevant proportion of total financial assets of the least wealthy households when overall exposure is considered. This raises the following question: if the distribution of wealth influences default incentives, how much of the increase in spreads is due to the increase in inequality, and how are risk and bond policies expected to evolve? Our objective is to build a model that sheds some light on this respect. As for the first question, we expect the deterioration of financial conditions to be the main driver for results up to 2011. It reduced output, increased debt and spreads, and generated higher inequality in government bond holdings, and more broadly, in net wealth. We are now interested on the feedback of this higher inequality into credit risk.

Proper Measure of Bank Exposure [TO DO]

The exercise is to assume a stark fall in government revenues that cannot be financed with further debt issuance, and given that government expenditure is rigid downwards, it implies a haircut on debt of x_d%. Note that this haircut must be such that additional post-default expenses, such us deposit insurance, have to be accounted for when balancing the government's budget constraint. We next see how this haircut affects the banks' balance sheet, bank by bank of our list.

There is a direct effect via direct bond holdings (corporate bonds should also suffer losses, check possible amount). Then there is an indirect effect via an increase in non-performing loans, since default negatively affects economic activity. We need to take this number from the data, and it implies a fall of x_L% in the value of loans. Banks' holding of stocks is minor. Thus we have an overall fall of x_A% in a bank's assets.

Suppose we keep the value of liabilities fixed (not obvious that this should be the case). How much would the new capital be, and how many resources are needed to go back to a capital-to-assets ratio fo 10%? Say 50% of junior debt is wiped out, and also 50% of senior debt (detailed classification still pending). If there still is need of resources, these will come from deposits. Thus we will have a ratio of deposits haircut to total deposits per bank, which we will average out or aggregate.

Next we go to the EFF survey and compute the effective haircut $Eff_d = \max\{0, \text{deposit haircut} - \text{insurance}\}$. We also need a measure of the effective insurance $Eff_{ins} = \min\{\text{insurance}, \text{deposit haircut}\}$. Aggregate this number for all households in the weighted sample, and take the ratio against total deposits of the sample. Use this ratio to the total deposits of the economy to see the actual fiscal burden. The initial haircut has to be such that this burden is covered. Note that this is an indirect selective default.

3. The Model

We develop a heterogeneous agents model in which households hold defaultable public debt as a savings vehicle. We consider a closed economy in which all output is consumed by households or purchased by the government and the entire stock of sovereign debt is held by domestic savers. Each period, the government observes the distribution of wealth and the state of the economy, and decides repayment and further debt issuances to maximize a utilitarian social welfare function. These decisions impact the savings behavior of agents and thus the distribution of wealth going forward.

The government collects taxes and issues short-term debt in order to pay for an exogenous random stream of government expenditures, in addition to serving its outstanding debt. At the beginning of each period, the government may revise its plans for lump-sum taxes as well as choose whether to repay its sovereign debt. This lack of commitment defines a problem between households and the benevolent government, and the solution is given by a Markov Perfect Equilibrium. The government's policies are a function of the aggregate state (which includes the distribution of wealth), and are taken as given by households. Then, each set of government policies defines a competitive equilibrium.

3.1 *The Competitive Equilibrium*

Let the idiosyncratic state be $s = (b, l)$ where b is bonds holdings and l is idiosyncratic endowment of income. Define the aggregate state to be $S = (A, \Lambda)$, where A is an aggregate income shock and $\Lambda = \Lambda(b, l)$ is the distribution of households, with a given law of motion $\Lambda' = \Psi_S(\Lambda)$.

A policy $\iota(S) = \{d(S), B'(S)\}$ for the government specifies bond issuance $B'(S)$ and the repayment policy $d(S)$. We assume that a default wipes out the entire stock of debt. Let $q(S)$ be the price of bonds.

The budget constraint of the government is

$$A \int l(1-\tau_{\tilde{\iota}} l^{\psi}) d\Lambda_l(l) + q(\mathbf{S}, \tilde{\iota}; \iota) B'_{\tilde{\iota}} = G(\mathbf{S}) + (1-d_{\tilde{\iota}})B + T$$

The household must decide savings b'_i and consumption c_i subject to a budget constraint and the restriction that public debt cannot be shorted

$$\begin{aligned} V(\mathbf{s}, \mathbf{S}; \tilde{\iota}, \iota) = \max_{c, b'} u(c) &+ \beta \mathbb{E}\left[V(\mathbf{s}', \mathbf{S}'; \iota, \iota) \mid \mathbf{s}, \mathbf{S}\right] \\ \text{subject to } c + q(\mathbf{S}, \tilde{\iota}; \iota) b' &= Al(1-\tau_{\tilde{\iota}}(l)) + (1-d_{\tilde{\iota}})b + T \\ &\tau_{\tilde{\iota}}(l) = 1 - \tau_{\tilde{\iota}} l^{-\psi} \\ &b' \geq 0 \\ &\text{laws of motion for } (\mathbf{s}, \mathbf{S}) \end{aligned}$$

Let $\mu(\mathbf{s}, \mathbf{S}, \iota)$ and $\chi(\mathbf{s}, \mathbf{S}, \iota)$ denote the multipliers associated to the borrowing constraint and to the non-negativity constraint respectively, and let $g_c(\mathbf{s}, \mathbf{S}, \iota)$ and $g_b(\mathbf{s}, \mathbf{S}, \iota)$ denote the consumption and savings optimal policies. Then the solution to the household's problem is characterized by two first-order conditions along with the envelope condition

$$\begin{aligned} [\text{c}]: &\quad u'(g_c(\mathbf{s}, \mathbf{S}, \iota)) = \mu(\mathbf{s}, \mathbf{S}, \iota) \\ [\text{b}']: &\quad q(\mathbf{S}, \iota)\mu(\mathbf{s}, \mathbf{S}, \iota) = \beta \mathbb{E} V_{b'}(s'_i, \mathbf{s}'; \iota, \iota) + \chi(\mathbf{s}, \mathbf{S}, \iota) \\ [\text{EC}]: &\quad d(\iota_d, \mathbf{S}) = 0 \iff V_b(\mathbf{s}, \mathbf{S}, \iota) = \mu(\mathbf{s}, \mathbf{S}, \iota) = u'(g_c(\mathbf{s}', \mathbf{S}', \iota)) \\ &\quad d(\iota_d, \mathbf{S}) = 1 \iff V_b(\mathbf{s}, \mathbf{S}, \iota) = 0 \end{aligned}$$

where V_b is the derivative of the household's value function with respect to initial bond holdings. The optimality conditions imply the following Euler equation

$$\begin{aligned} &u'(Al(1-\tau_{\tilde{\iota}}(l)) + T + b(1-d(\iota_d, \mathbf{s})) - q g_b(\mathbf{s}, \mathbf{S}, \iota)) \geq \\ &\frac{1}{q}\beta \mathbb{E}\left[u'(A'l'(1-\tau_{\iota}(l')) - T + g_b(\mathbf{s}, \mathbf{S}, \iota) - q' g_b(\mathbf{s}', \mathbf{S}', \iota)) \mid \mathbf{s}, \mathbf{S}\right] \end{aligned}$$

with equality when the borrowing contraint is slack. Households will then need to forecast next period's prices for each possible value of the aggregate state.

Definition: Given a government policy ι, a **Competitive Equilibrium** has value and policy functions

$\{V(s,S), g_c(s,S), g_b(s,S)\}$, prices $\{q(\mathbf{S}, \iota), w(\mathbf{S}, \iota)\}$, distribution $\{\Lambda_{\mathbf{S}}(\mathbf{s})\}$, and a law of motion for the distribution such that

i. Given prices, (g_c, g_b) solve the household's problem *ii*. Competitive factor markets: $w = A$

iii. The government's policy satisfies its budget constraint

iv. Bond market clear

$$B'_\iota(\mathbf{S}) = \int g_b(\mathbf{s}, \mathbf{S}, \iota) d\Lambda_{\mathbf{S}}(\mathbf{s})$$

v. The law of motion satisfies

$$Q_\iota((b,l), \mathcal{B} \times \mathcal{L}; \mathbf{S}) = \mathbb{I}_{\{g_b(\mathbf{s},\mathbf{S},\iota) \in \mathcal{B}\}} \int_{\mathcal{L}} F_l(dl'|l)$$
$$\Lambda_{\mathbf{S}'}(\mathcal{B} \times \mathcal{L}; \iota) = \Psi_S(\Lambda) = \int Q_\iota((b,l), \mathcal{B} \times \mathcal{L}; \mathbf{S}) d\Lambda_{\mathbf{S}}(\mathbf{s})$$

3.2 *The Government's Problem*

Each period, the government chooses the current policy ι_d taking as given all future strategies, including the future policy ι. The government's objective is to

$$\max_{\iota_d} \mathcal{W}_t(\mathbf{s}_t, \iota_d, \iota) = \int V(s^i_t, \mathbf{s}_t; \iota_d, \iota) d\Lambda_t(s^i_t) \tag{1}$$

where, critically, $V(s^i_t, \mathbf{s}_t; \iota_d, \iota)$ is the household's optimal value function in the competitive equilibrium that arises under (ι_d, ι).

Expanding the household's value function, we can write the government's value as

$$\begin{aligned} \mathcal{W}_t(\mathbf{s}_t, \iota_d, \iota) &= \int u\left(c(s^i_t, \mathbf{s}_t; \iota_d, \iota)\right) + \beta \mathbb{E}_t\left[V(s^i_{t+1}, \mathbf{s}_{t+1}; \iota, \iota)\right] d\Lambda_t(s^i_t) \\ &= \int u\left(c(s^i_t, \mathbf{s}_t; \iota_d, \iota)\right) d\Lambda_t(s^i_t) + \beta \mathbb{E}_t\left[\int \left(\int V(s^i_{t+1}, \mathbf{s}_{t+1}; \iota, \iota) d\Lambda_{t+1}(s^i_{t+1})\right) d\Lambda_t(s^i_t)\right] \\ &= \int u\left(c(s^i_t, \mathbf{s}_t; \iota_d, \iota)\right) d\Lambda_t(s^i_t) + \beta \mathbb{E}_t\left[\int V(s^i_{t+1}, \mathbf{s}_{t+1}; \iota, \iota) d\Psi(\Lambda_t(s^i_{t+1}), \mathbf{s}_t)\right] \\ &= \int u\left(c(s^i_t, \mathbf{s}_t; \iota_d, \iota)\right) d\Lambda_t(s^i_t) + \beta \mathbb{E}_t\left[\mathcal{W}(\mathbf{s}_{t+1}, \iota, \iota)\right] \end{aligned} \tag{2}$$

where the second equality used iterated expectations (conditioning on Λ_{t+1}) and the third used that there is nothing random about Λ_{t+1} conditional on Λ_t and $\mathbf{s}_t$.

For a government policy $\iota^\star$ to be a part of a Markov Perfect equilibrium, it has to leave the government no profitable deviations in every possible state. By the one-shot principle, the government must weakly prefer to abide by the policy than to do something else, *even when* doing something else does not change the way in which agents form expectations about the government's future behavior.

Definition A *Markov Perfect Equilibrium* of the economy is a government policy $\iota^\star$ along with a competitive equilibrium such that

(i) The equilibrium is a competitive equilibrium under $(\iota^\star, \iota^\star)$ as defined above.

(ii) The policy $\iota^\star$ leaves the government no profitable deviations

$$\mathcal{W}(\mathbf{s}, \iota^*, \iota^*) = \max_{d,\tau} \int \left\{ \max_{b_i' \geq 0} u\left(Al(1-\tau_{\iota^*}(l)) + T + b(1-d) - q(d,\tau,\mathbf{s})b' - \tau\right) \right\} d\Lambda \tag{3}$$
$$+ \beta \mathbb{E}[\mathcal{W}(\mathbf{s}', \iota^*, \iota^*)]$$

The equilibrium of this economy is one where households correctly infer the probability of default given the government's rule and, given that inference, the government finds it optimal to default in states that carry the correct probability. In this model where domestic agents price debt, risk aversion and the correlation between consumption and default are reflected in the price of debt. In other words, the stochastic discount factor that prices government debt is endogenous. This is very different from models where foreign risk neutral creditors force the price of debt to only reflect the probability of default (cf. Arellano, 2008, and others).

4. Solution Method

When solving their problem, both the government and households need to keep track of the distribution $\Lambda(b,l)$. An efficient and consistent way to avoid the infinite dimensionality problem is to assume that both types of agents are bounded rational, and believe that the distribution can be parametrized by a finite set of moments $\Lambda \approx F(\mathcal{M}(\Lambda))$. In particular, we approximate the distribution of (b,l) to be joint $\ln\mathcal{N}(\mu,\Sigma)$, where μ is the vector of means and Σ is the covariance matrix. This approach is consistent because (μ,Σ) now enters the state space of the agents, its law of motion can be directly computed from optimal policies, so that they are embeded in the solution of the problem of the household. That is, in contrast to Krusell and Smith (1998), there is no need to iterate over laws of motion. It is also consistent because the government shares the same beliefs when making the optimal choices.

The solution method for the household's problem is not novel. Winberry (2016) and Reiter (2010) have already suggested solution methods with similar characteristics, but in a context without aggregate shocks, or with perturbations around the steady state. Our method, however, goes a step further by avoiding a linear approximation of the solution and by allowing to solve not just for the households problem but for the entire MPE.

We then check the accuracy of the solution method according to several measures measures. The first set of measures is used to evaluate the accuracy of assuming that the current distribution is lognormal. Households use this to forecast B', V_B' and $V_{b,l}'$. The government will use this assumption to evaluate social welfare. The second set of measures is used to evaluate the accuracy of assuming that next period's distribution is also lognormal. This is just used by households to forecast q'. A detailed specification of these measures will be presented after further describing the solution method.

In what follows we describe in more detail the solution method. Under the lognormal assumption, given a mean B and variance V_B of a variable in levels, the mean μ_b and variance Σ_b of the variable in logs are given by

$$\Sigma_b = \ln\left(1 + \frac{V_b}{B^2}\right)$$
$$\mu_B = \ln(B) - 0.5\Sigma_b$$

In addition, the following equation gives the relation between the covariance of the variables in levels and the variance and covariance of the variables in logs:

$$V_{b,l} = [e^{\Sigma_{b,l}} - 1]\exp\left\{\mu_b + \mu_l + 0.5(\Sigma_b + \Sigma_l)\right\}$$

Also note that at any point in time, μ_l and Σ_l are constant. With this in mind, let the new aggregate state be $S = (A, B, V_B, V_{b,l})$. Under no government deviation, the household problem is:

$$V^{(n)}(\mathbf{s}, \mathbf{S}, q; \iota) = \max_{c,b'} u(c) + \beta\mathbb{E}\left[V^{(n-1)}(b', l', B', V'_B, V'_{b,l}, q'; \iota)\right]$$
$$\text{s.t.} \bullet\ c + qb' = Al(1 - \tau_{\bar{\iota}}(\mathbf{S})l) + T + b(1 - d_\iota(\mathbf{S}))$$
$$\bullet\ b' \geq 0$$
$$\bullet\ B' = \int g_b^{(n-1)}(\mathbf{s}, \mathbf{S}, \iota) dF(\mu, \Sigma)$$
$$\bullet\ V'_B = \int \left(g_b^{(n-1)}(\mathbf{s}, \mathbf{S}, \iota) - B'\right) dF(\mu, \Sigma)$$
$$\bullet\ \Sigma_{b,l} = \int (\ln(b') - \mu'_b)(\ln(l') - \mu'_l) d\mathcal{N}(\mu, \Sigma) = \int (\ln(b') - \mu_b)\rho_l \ln(l) d\mathcal{N}(\mu, \Sigma)$$
$$\bullet\ B''_\iota = \int g_b^{(n-1)}(b', l', B', V'_B, V'_{b,l}, q', \iota) dF(\mu'_B, \Sigma')$$
$$\bullet\ \tau_\iota = \left[L - \frac{g(\mathbf{S}) + B(1-d) + T - B'_\iota(\mathbf{S})q}{A}\right] \frac{1}{\int l^{1-\psi} d\Lambda_l(l)}$$

We use the Envelope Condition Method (ECM) to solve for the household's problem. The algorithm is as follows:

(i) At iteration n, we need policy $g_b^{(n-1)}(\mathbf{s}, \mathbf{S}, \iota)$ and value $V_b^{(n-1)}(\mathbf{s}, \mathbf{S}, \iota)$.

  (a) Find unconstrained $\tilde{g}_c(\mathbf{s}, \mathbf{S}, \iota) = (u')^{-1}\left(V_b^{(n-1)}(\mathbf{s}, \mathbf{S}, \iota)\right)$

  (b) Get $\tilde{g}_b(\mathbf{s}, \mathbf{S}, \iota)$ from budget constraint.

  (c) $g_b(\mathbf{s}, \mathbf{S}, \iota) = \max\{0, \tilde{g}_b(\mathbf{s}, \mathbf{S}, \iota)\}$.

  (d) Get $g_c(\mathbf{s}, \mathbf{S}, \iota)$ from budget constraint.

(ii) Update the guess

$$\text{[Constrained]}: \qquad V_b^{(n)}(\mathbf{s}, \mathbf{S}, \iota) = u'(g_c(\mathbf{s}, \mathbf{S}, \iota))$$
$$\text{[Unconstrained]}: \qquad V_b^{(n)}(\mathbf{s}, \mathbf{S}, \iota) = \frac{\beta}{q(\mathbf{S}, \iota)} \mathbb{E}\left[V_b^{(n-1)}(\tilde{g}_b(\mathbf{s}, \mathbf{S}, \iota), l', B', V'_B, q')\right]$$

(iii) Iterate until convergence

Once we have solved the problem of the household, we have to extend to allow for a one-period deviation $\tilde{\iota}$. Then we can solve the problem of the government as

$$W(\mathbf{S}, \tilde{\iota}, \iota) = \max_{\tilde{\iota}} \int V(\mathbf{s}, \mathbf{S}, \tilde{\iota}, \iota) dF(\mu, \Sigma)$$

A Markov Perfect equilibrium is a policy $\iota^{\star}$ such that

$$\iota^{\star} \in \arg\max_{\iota} W(\mathbf{S}, \iota, \iota^{\star})$$

Accuracy Measures We simulate the model starting from the solution to the case where the TFP remains constant at the mean level, and drop the first 200 periods. Let $\tilde{\Lambda}_t$ be the simulated histogram of (b, l), and let $F(\mu_t, \Sigma_t)$ be the lognormal distribution. The simulation delivers $\{A_t, B_t, \tilde{\Lambda}_t, \mu_t, \Sigma_t, d_t\}_{t=1}^{T}$. Note that $\mathcal{M}(\tilde{\Lambda}_t) = (\mu_t, \Sigma_t)$ come from $\tilde{\Lambda}_t$. Our first measure is to check if the non-centered moments of $\tilde{\Lambda}_t$ are similar to the same moments generated by $F(\mathcal{M}(\tilde{\Lambda}_t))$. The second measure is to compare market clearing r_t and B_{t+1} under $\tilde{\Lambda}_t$ vs $\mathcal{M}(\tilde{\Lambda}_t) = (\mu_t, \Sigma_t)$. These two measures convey the goodness of fit for the current distribution. Our third measure consists on comparing $E_t[b_{t+1}^n]$ for $n = \{2, 3, 4\}$, when assuming $\tilde{\Lambda}_t$ vs when assuming $F(\mathcal{M}(\tilde{\Lambda}_t))$. This gives us an idea on wether the lognormality assumption distorts next period's distribution. The fourth measure is to compute the moments of $F(\hat{\mu}_{t+1}, \hat{\sigma}_{t+1})$, where $(\hat{\mu}_{t+1}, \hat{\sigma}_{t+1})$ are the moments implied by household's policy when we assumed $F(\mathcal{M}(\tilde{\Lambda}_t))$ at time t. This measure is because the law of motion implied by households' policies may not preserve lognormality. The last measure is to find next period's market clearing price under $F(\hat{\mu}_{t+1}, \hat{\sigma}_{t+1})$, and compare that price with the observed one. This last measure directly gives us an idea about how good the household's approximation for q' is.

5. Quantitative Results

In what follows we describe the quantitative results for both, the steady state equilibrium and the competitive equilibrium. The former already provides significant economic intuition of the forces underlying in the model. The latter is a first step to solving for the MPE in the economy with aggregate shocks and provides information about the goodness of our solution method.

5.1 The Steady State and Transition Dynamics

5.1.1 Definition and Results of the Steady State

Given a government policy τ, a **Stationary RCE** has value and policy functions $\{V, g_c, g_b\}$, prices $\{q(\tau), w\}$, and a measure $\Lambda^*(\tau)$ s.t.

i. Given prices, households solve their problem

ii. Factors of production are paid their marginal product $w = A$

iii. The government's policy satisfies its budget constraint

$$B = \frac{\tau - T - g}{1 - q}$$

iv. Bond markets clear

$$B = \int g_b(s_i; \tau) d\Lambda_\tau$$

v. For all $\mathcal{B} \times \mathcal{L} \in \Sigma_{S_i}$, the invariant measure Λ^*_τ satisfies

$$Q_\tau((b,l), \mathcal{B} \times \mathcal{L}; \tau) = \mathbb{I}_{\{g_b(b,l;\tau) \in \mathcal{B}\}} \int_{\mathcal{L}} F_l(dl'|l)$$

$$\Lambda^*_\tau(\mathcal{B} \times \mathcal{L}) = \int_{s_i \in S_i} Q_\tau(s_i, \mathcal{B} \times \mathcal{L}) d\Lambda^*_\tau$$

On this section we start by exploring the optimal quantity of debt by comparing steady-states and highlighting the differences across them that make some preferred to others. We solve the model assuming no default[9]. Note that the level of debt is pin down by $B = \frac{\tau - T - g}{1-q}$, where q is an equilibrium price. The level of expenditures g is a parameter, and τ is being moved to compute different SS equilibria. We then show how the economy moves in transition after the government changes the amount of debt issued or imposes a haircut on some of its outstanding debt.

We assume a symmetric distribution for the endowment of income, and discretize it using Tauchen's method. The risk aversion parameter γ is taken from standard values used in the literature. We normalize TFP to unity. The calibration for the income process is still preliminary. Guvenen (2009) finds that the persistance of labor income in the US ranges from 0.80 to 0.99 depending on the sample and on the assumed underlying process. Floden and Lindé (2001) estimate a process for the log of wages for the US, assuming a persistent plus a trainsitory shock, and get an autocorrelation coefficient of 0.91, and an implied variance of labor income of 0.3[10]. In addition, Pijoan-Mas and Sanchez-Marcos (2010) analyze the Spanish labor market for the years 1985 until 2000 using a cross-sectional survey data, finding that the variance of the log of wages is 0.25. Our data comes from "Encuesta Financiera de las Familias" (EFF), a survey on Spanish households' income and assets elaborated by the Bank of Spain, and it is available for the years 2002, 2005, 2008, 2011 and 2014. This is a cross-sectional database, but households are asked for their income during the previous natural year. As detailed in the appendix, we construct a variable on pre-tax labor income that includes all possible types of income for an agent active in the labor market, and we aggregate for all aggents in a household. We obtain an average variance of the log of labor income of 0.34, by averaging the cross-sectional variance for the years 2001, 2002, 2004 and 2005.

[9]Default is a shock on aggregate variables which moves the economy away from the steady state. Moreover, if we take the stand that steady states are a depiction of 'normal times' then it make sense to think about them as times when default risk is low.

[10]See appendix for details, we are still working on the estimation.

Consistent with the literature, we fixed $\rho_l = 0.91$ and backed out $\sigma_l = 0.24$ so as to get the variance observed in the data. To calibrate the tax progressivity parameter ψ, we compute a percentage change of -10% for the pre-tax vs post-tax Gini index on total income. Households' total income can be obtained from the panel data survey "Encuesta de Condiciones de Vida" for the years 2008-2017, as the sum of labor income, capital income and social transfers[11]. This survey does not provide a separate measure for labor income and capital income, but in contrast to the EFF, it does allow us to compare a pre-tax and post-tax Gini index. The implicit assumption here is that taxation on capital and labor income are similar. Finally, government final consumption expenditure, domestic debt and transfers were obtained from publicly available sources. The latter are minor social transfers, in particular, *Subsidies, payable* and *Other current transfers, payable* from Eurostat. Since in our model we have a closed economy without investment in productive capital, our measure of output is households plus general government final consumption expenditure. We chose $\beta = 0.84$ as to match an average interest rate of 4.6% when $B = 0.34$ in the model.

Parameter	Value	Description	Source
γ	2.0	Risk aversion	Standard
ρ_l	0.91	Persistence of Income	Literature and preliminary estimations
σ_l	0.23	S.D. of Innovation	Match variance of labor income
A	1.0	Aggregate TFP	Normalization
g	0.23	Government spending	Eurostat (1995-2011)
T	0.035	Government Transfers	Eurostat (1995-2011)
B	0.34	Domestic Debt	Eurostat and Tesoro Publico (2002-2011)
r	0.046	Government Bond Yield	Eurostat (1995-2011)
β	0.84	Discount factor	Match interest rate at B
ψ	0.10	Tax Progressivity	Match $\Delta\%$ in Gini (2008-2011)

Figure (9) compares the model economy across steady-states indexed by the quantity of debt (output is constant and normalized to one). Equilibria with more debt feature higher taxes (i.e. lower τ) and a higher interest rate. Welfare, measured as the utilitarian objective, peaks just above 28% of GDP in government debt. Several forces are at play, explaining the concavity of social welfare.

[11] The 2008-2011 average pre-tax Gini index was 0.4 and the average post-tax Gini index was 0.36. As a comparison, the Gini index of income post taxes and transfers obtained from the OECD for the years 2007-2015 is 0.34.

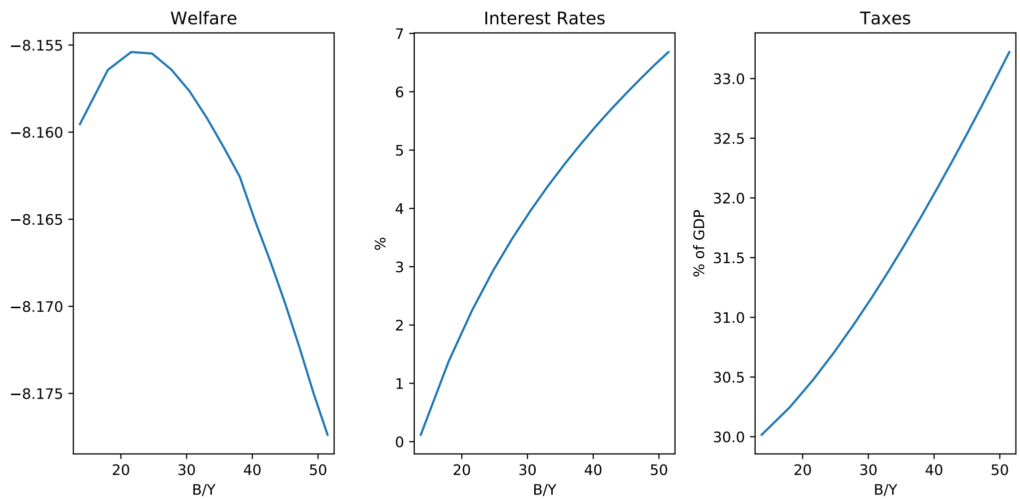


FIGURE 9: WELFARE ACROSS STEADY-STATES

With higher debt, the government mechanically needs to collect higher taxes. A higher interest rate is needed for demand to match the higher supply, shifting the distribution of bond holdings to the right. On the one side, wealthy households get net benefits due to higher accumulation of bonds, as well as a higher return on these bonds, in addition of being able to move away from the borrowing constraint. On the other side, poorer households are not able to accumulate as many assets, incurring into a higher tax burden and net loss. Overall, when debt is too low there are social benefits of increasing, but as debt passes 30% and tax burdens increase, less wealthy households start dominating the average social welfare. Nonetheless, it is worth noticing that the scale of welfare on the graph is quite small.

These are similar results as the ones obtained by Aiyagari and McGrattan (1998). They find that changes in aggregate bonds does not entail significant changes in social welfare, and thus the actual level of debt of the US government, even though not the optimal one, does not imply a significant social loss. The reason behind this result is that the distribution of bond holdings changes with variations in the government's policy. That is, even though households in a particular point of the distribution have a loss in welfare when aggregate bonds increase, the distribution will shift to the right, giving more weight to wealthier households. These two effects net out, yielding an almost flat social welfare.

5.1.2 Transition Dynamics Under No Government Choice

On this section we compute the transition dynamics of a 50% haircut on a SS with 102% debt, with gradual recovery. Even though at no point there is default risk, the objective is to know whether the government would ever want to default.

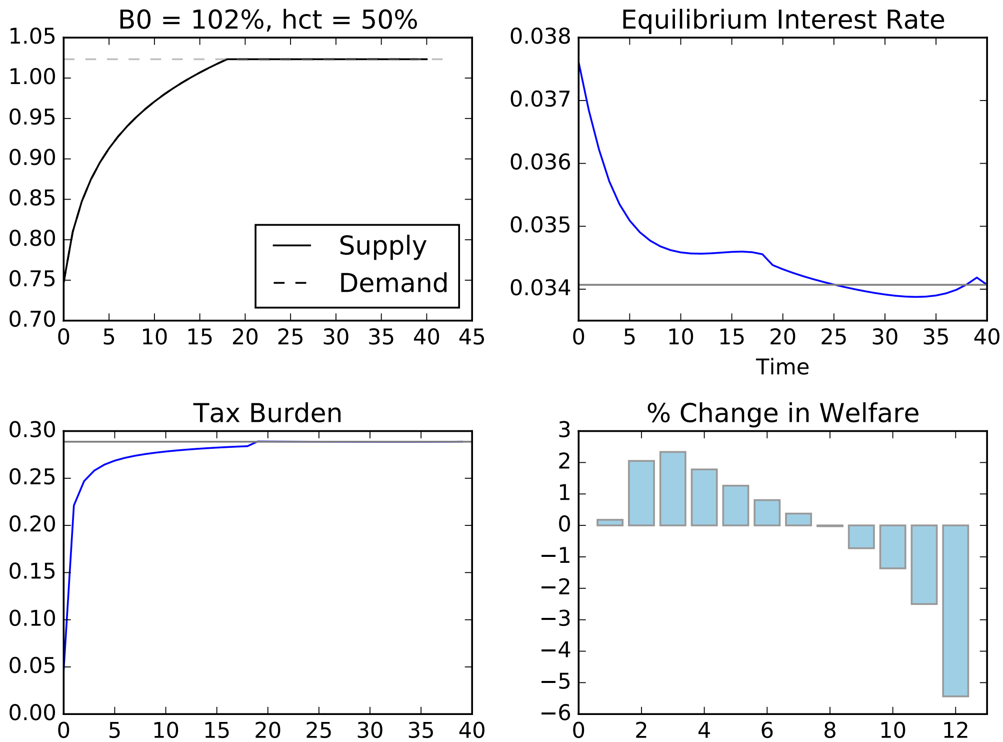


FIGURE 10: TRANSITION DYNAMIC UPON HAIRCUT

There is a short-term reduction in taxes and increase in the interest rate. The latter is because the suddent fall in assets significantly affects the desire to save, up to the point where there is excess supply of bonds. Thus, for markets to clear it must be that interest rates are high enough. Over the long term, these variables revert to their original values as debt returns to the steady state level. Overall, there are heterogeneous effects on welfare. Households below the median see their welfare increased, while those above the median are worse off upon the haircut. When aggregating, the policy is socially benefitial, since social welfare increases by 0.20%.

Figure 11 below shows households policy and the evolution of the distribution. The less wealthy households leverage on the temporary reduction in taxes by increasing savings and consumption in the short term. In contrast, wealthier households reduce savings and consumption. The distribution of wealth becomes more equal for a while, but it eventually returns to its original shape.

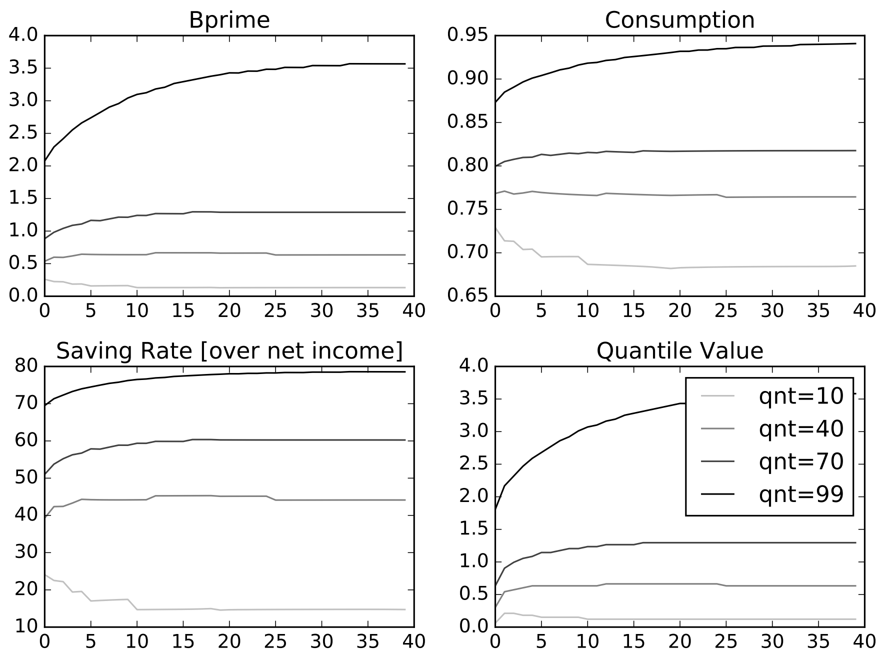


FIGURE 11: TRANSITION DYNAMIC: POLICIES

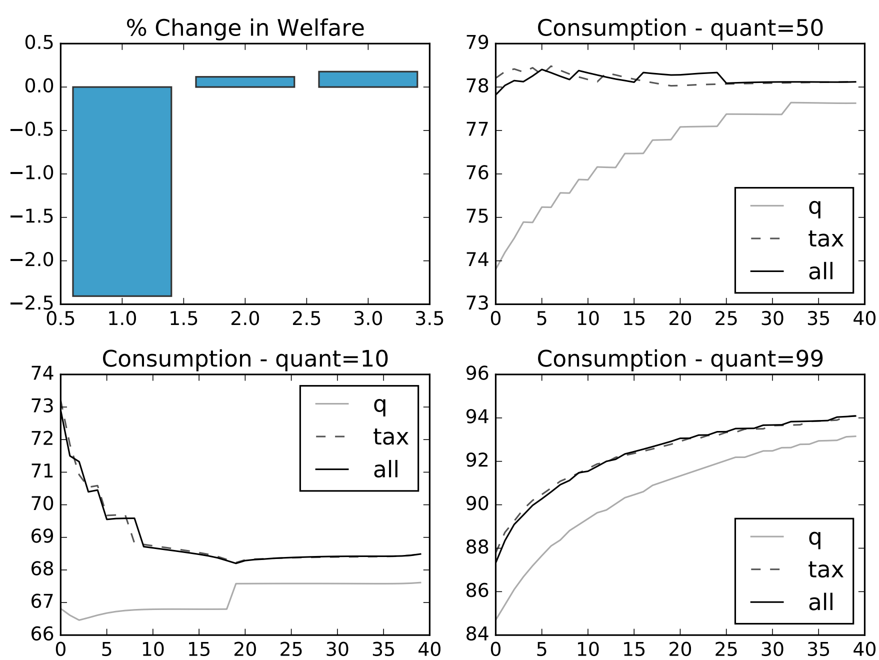


FIGURE 12: TRANSITION DYNAMIC: DECOMPOSITION OF EFFECTS

Finally, figure 12 above decomposes the effects that the change in the path of taxes and the path of prices have on social welfare. To see the effects of taxes, we keep prices at their original levels and compute welfare if only taxes change, taking the path seen in the prior dynamics. Similarly, to compute the effect of prices, we keep taxes fixed and allow for the observed path in prices. If we only consider the change in prices, welfare would dramatically fall because taxes would be too high to compensate for the increase in interest rates. If we keep prices fixed, the short-term fall in taxes, and consequent increase in consumption for the lowest quantiles effectively increases social welfare. When considering changes in both variables social welfare is slightly higher, denoting a positive interactive effect.

5.2 *Competitive Equilibrium*

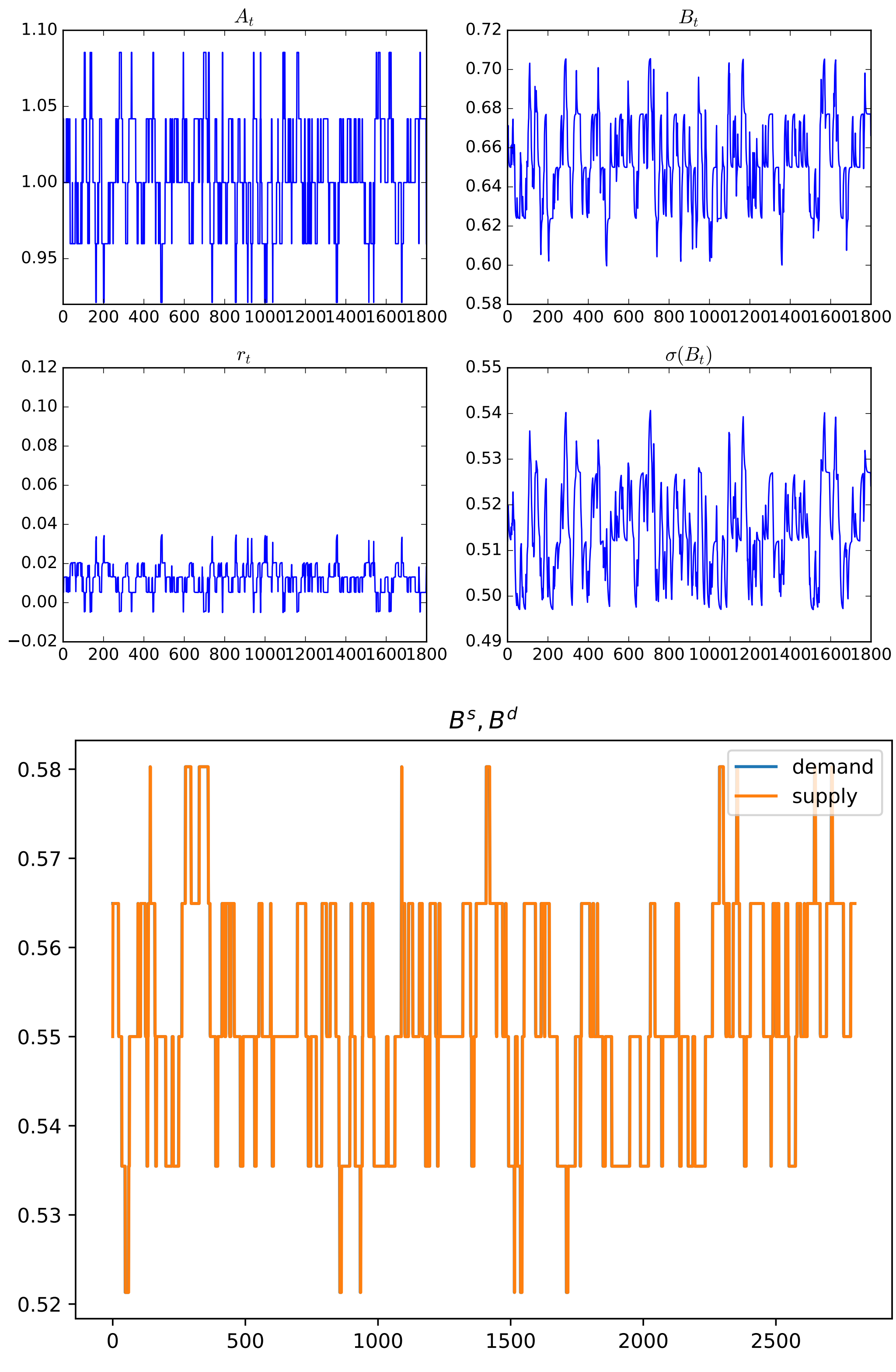


FIGURE 13: SIMULATION AND MARKET CLEARING

Figure (13) above is the result of a simulation for A_t. We assume households believe Σ is diagonal. We also assume the government's policy is such that $\frac{B'}{Y}$ is kept constant, there is no default and the government's budget constraint is satisfied. Therefore, if markets clear, the evolution of B must follow one-to-one the evolution of A. We also show the market clearing interest rate r_t and the cross-sectional deviation of bond holdings $\sigma(B_t)$. The latter shows variations but not in high magnitudes. Also, the real rate is quite low. Both facts can be explained by high demand for bond holdings due to precautionary savings motive. This is shown in the last plot, where at some points demand is higher than supply, indicating we need to allow for even lower interest rates.

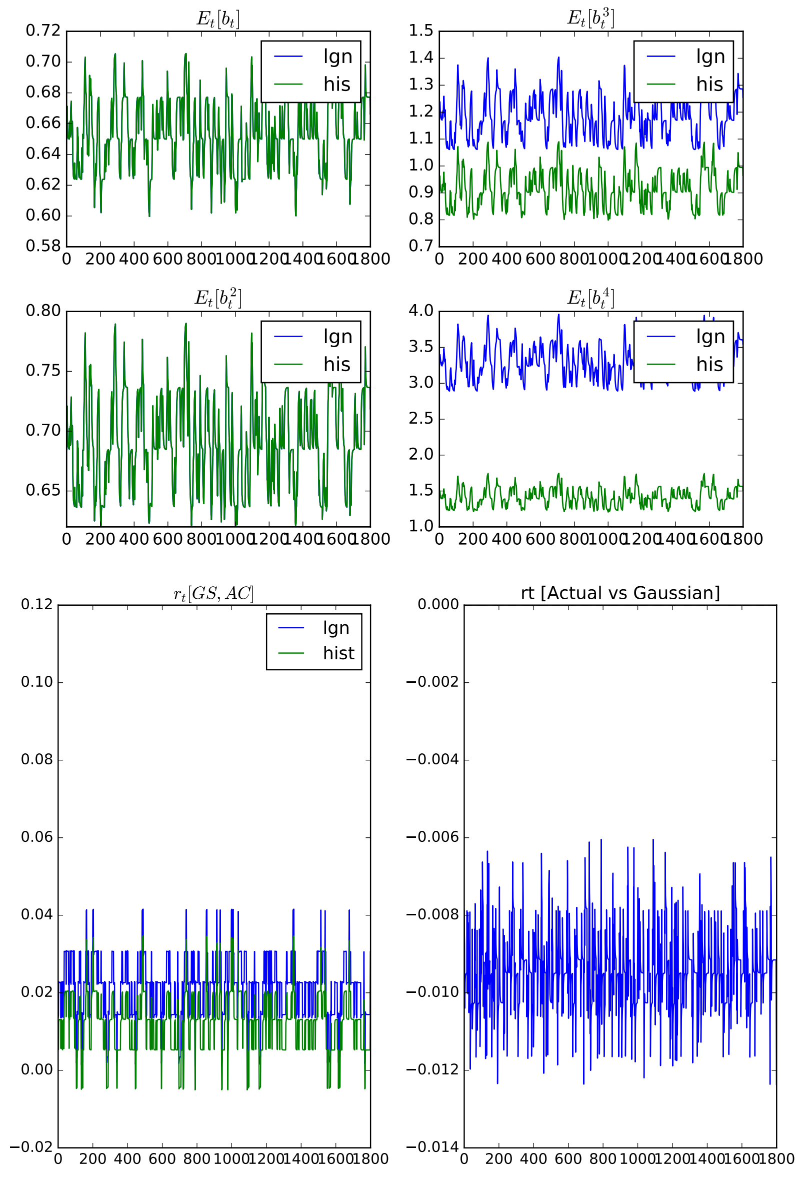


FIGURE 14: GOODNESS OF FIT - CURRENT DISTRIBUTION

Figure (14) above shows the goodness of fit for the current distribution. The set of the first four plots compares non-centered moments for the simulated histogram $\tilde{\Lambda}_t$ vs. the Lognormal distribution

$F(\mathcal{M}(\tilde{\Lambda}_t))$. The first four plots show how good the approximation for the current distribution is. The first two moments are the same by construction, and the last two moments are almost the same in levels, and have a correlation of XX %. The second set of plots show the market clearing interest rate and quantity of bonds, comparing $\tilde{\Lambda}_t$ vs. $F(\mathcal{M}(\tilde{\Lambda}_t))$. The equilibrium level of bonds take almost the same values, and the interest rates some deviations on limited occassions. Overall, the assumption of current lognormality seems to be a good approximation. This was used by the household to forecast (B', V_B'), and by the government to compute social welfare.

The next step is to analyze the implications of assuming $F(\mathcal{M}(\tilde{\Lambda}_t))$ on next period's distribution. This is done in steps in figure (15) below. The first set of four plots presents the implied moments for next period. This is done by integrating the bond policies when assuming $\tilde{\Lambda}_t$ vs. $F(\mathcal{M}(\tilde{\Lambda}_t))$. Once again, they take almost the same values and the correlation is of XX %. However, the law of motion implied by the policies does not guarantee that lognormality is preserved. On this respect, the next set of plots goes a step further and computes the moments under $\tilde{\Lambda}_{t+1}$ vs. $F(\hat{\mu}_{t+1}, \hat{\sigma}_{t+1})$, where $(\hat{\mu}_{t+1}, \hat{\sigma}_{t+1})$ are computed by integrating the policies under $F(\mathcal{M}(\tilde{\Lambda}_t))$. The computed moments take similar values under both distributions with a correlation of XX %.

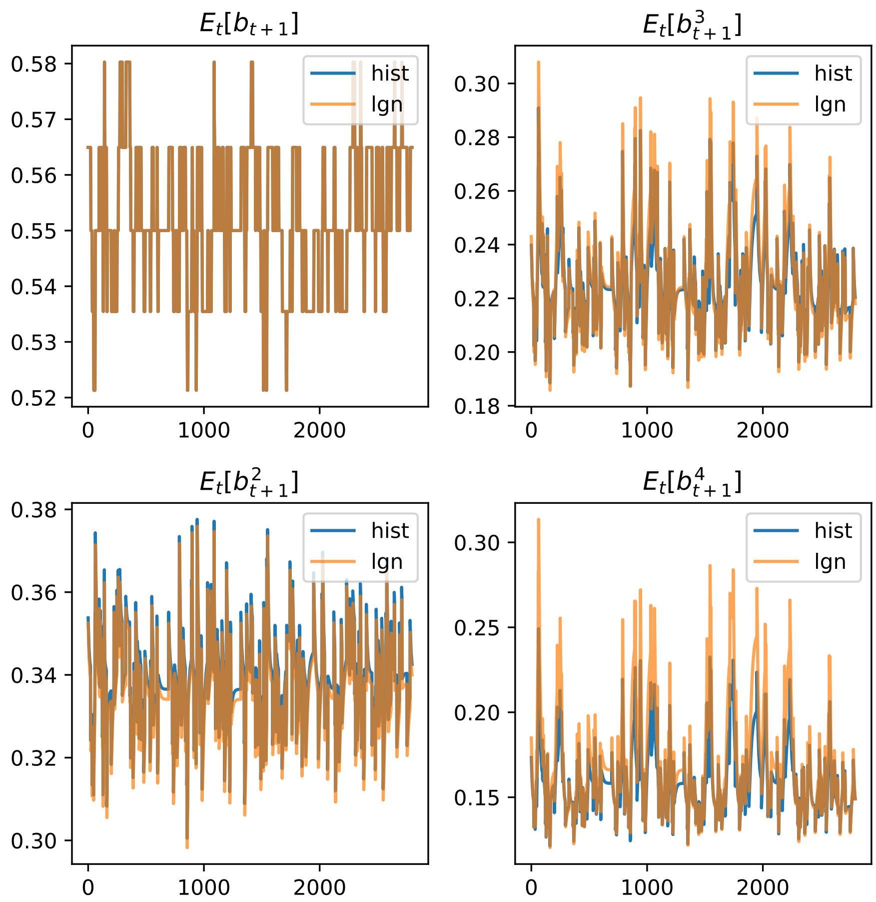


FIGURE 15: GOODNESS OF FIT - NEXT PERIOD'S DISTRIBUTION

Lastly, we need to check how good the household's approximation of q' is. Since we showed that market clearing outcomes do not vary much between $\tilde{\Lambda}_t$ and $F(\mathcal{M}(\tilde{\Lambda}_t))$, and we also showed that next period's distribution remains fairly lognormal, we expect the approximation $\hat{q}'$ to be close to the actual q'.

6. Concluding Remarks

We have built a model in which the distribution of households across sovereign debt positions has non trivial effects on the choice of sovereign debt policy. Our preliminary results suggest that, first, a comparison across steady-states with different levels of debt is likely to be insufficient when assessing policies and, second, that the distribution plays a central role in determining debt policy incentives, both across steady-states and when considering transitions.

We have also proposed a way to solve for the MPE in an economy with aggregate shocks, without the need to iterate over the assumed laws of motion. Furthermore, the assumptions made regarding the distribution of households is internally consistent in the sense that it is taken by both, households and the government, to make optimal choices. This solution method appears to be efficient and very accurate when restricted to a competitive equilibrium without default. The next step would be to add default and to solve for the full MPE.

Even though at this point we can only speculate about what the full MPE will look like, we believe that the forces highlighted here will be present and will determine the evolution of the economy. We expect to learn much about what kinds of states feature default in the equilibrium policy, to shed light on the internal costs of default question that motivates our research, and finally to be able to link our findings to the empirical features of the Spanish crisis.

7. Appendix

7.1 *Appendix 1: Details on the Spanish Banking System*

The banking system in Spain is mainly divided between commercial banks and savings banks. As of 2011, commercial banks accounted for 50% of total non-financial sector's (NFS) deposits, the other 50% being the savings banks[12]. The main banks, that is, Banco Santander, BBVA, Banco Popular, Banco Sabadell, Banesto and Bankinter, accounted for 84% of that 50%. The remaining 16% are commercial banks that hold less than 1% of the system's NFS deposits.

As for the savings banks, Bankia, Caixabank, NCG, Banco Mare Nostrum, BBK Kutxa, Ibercaja and Unicaja hold 67% of NFS deposits in savings banks[13]. NCG was intervened and adquired by the central government in 2011, and it was privatized in Nov-2013. Smaller savings banks such as CatalunyaCaixa, Unnim, Banco CAM and Banca Civica were either abosrbed by the system, or intervened by the government. The rest of the savings banks are small compared to the banking system.

All in all, our final list of banks (in order of 2011 deposits) are: BBVA, Santander, Bankia, Caixa-

[12] Data was obtained from the Bank of Spain, the Asociacion Espanola de Banca and the banks' annual reports.
[13] Data obtained from the banks' annual reports.

bank, Banesto, Banco Popular, Banco Sabadell, Banco Mare Nostrum, BBK Kutxa, Ibercaja and Unicaja. They account for 70% of NFS deposits. We exclude NCG just to focus on private banks.

7.2 *Appendix 2: Government's Intervention on the Spanish Banking System*

In 2008, the Central Government created a €30bn fund to finance Spanish banks (€19bn was used), as well as a mechanism to guarantee instruments issued by domestic banks of up to €100bn (almost €70bn was actually used). It also increased deposit insurance from €20, 000 to €100, 000.

In 2009 it created the Fund for the Orderly Restructuring of the Banking Sector (FROB I) of €9bn, which was managed by 5 representatives of the Bank of Spain and other 3 representatives from the Deposits Insurance Fund. The central government provided €6.7bn for its creation. The objective of FROB I were (i) to oversee the reestructuring process of troubled institutions; and (ii) to help finance mergers within the banking sector. By Spanish law, any troubled bank should first seek a private solution. If that was not possible, it could ask the Insurance Deposits Fund for financing, should the reestructuring or bank resolution plan be approved. The fund would be financed by the FROB. In case the plan was not approved, the Bank of Spain could take over the bank, and the FROB would manage it.

In 2011 FROB I was expanded to FROB II. It allowed the fund to buy common stocks of banks that did not satisfy capital requirements. Then FROB would proportionally participate in the management of that bank. Adquisitions could only be temporal. Overall, FROB I invested €9.7bn and FROB invested €4.7bn.

7.3 *Appendix 3: Measurement of the Income Process*

To construct the variable of labor income per household we consider each agent of the household separately and then add them up. The possible sources of income are (i) compensation and profits from a household-owned firm, (ii) wages from an outside firm, (iii) government unemployment insurance, (iv) pensions (both public and private) and (v) private insurance. We define the head of household to be the agent with highest income coming from either (i) or (ii) on the reference year.

Our estimation strategy for the parameters of the income process is based on Floden and Lindé (2001). We assume the following underlying process for labor income

$$y^{i,t} = c_i + z_{i,t} + \xi_{i,t}$$
$$z_{i,t} = \rho z_{i,t-1} + \epsilon_{i,t}$$

where c_i is a household fixed effect, $\xi_{i,t} \sim \mathcal{N}(0, \sigma_\xi)$, $\epsilon_{i,t} \sim \mathcal{N}(0, \sigma_\epsilon)$ and both innovations are i.i.d.

Since we do not have a proper panel, we follow the authors and estimate the fixed effects c_i using household-specific variables, and we then obtain our final measure of labor income

$$\tilde{y}_{i,t} = y_{i,t} - \beta \mathbf{X}_{i,t}$$

where $\mathbf{X}_{i,t}$ includes age, sex, marital status, and level of education of the head of household.

We would then estimate $\{\rho, \sigma_{\xi}, \sigma_{\epsilon}\}$ by GMM using the following moment restrictions

$$\mathbb{E}[\tilde{y}_{i,t}^2] - \frac{\sigma_\epsilon^2}{1-\rho^2} - \sigma_\xi^2 = 0$$
$$\mathbb{E}[\tilde{y}_{i,t}\tilde{y}_{i,t-j}] - \rho^j \frac{\sigma_\epsilon^2}{1-\rho^2} = 0$$

Focusing on the pre-crisis, we have labor income data for the years 2001, 2002, 2004 and 2005. This gives us 6 moment conditions for 3 unkowns.

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