Uncertainty Premia, Sovereign Default Risk, and State-Contingent Debt*

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Abstract

We study the pricing, design, and desirability of sovereign state-contingent debt instruments (SCDIs). Using a sovereign default model with lenders who fear model misspecification, we find that the commonly used threshold bond structure leads to welfare losses for the government. While this bond would be beneficial when facing rational-expectations lenders, its threshold structure increases the variance of promised returns, which robust lenders dislike. SCDIs can still be welfare improving when facing robust lenders when designed optimally. Our findings tie the lack of popularity of sovereign state-contingent debt instruments to the particular design used thus far.

JEL Classification E43, E44, F34, G12, H63, O16

Keywords Sovereign debt, default risk, state-contingent debt instruments, robust control, ambiguity premia

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1 Introduction

The recent European sovereign debt crises and the increase in public debt levels after the COVID-19 shock have brought proposals for state-contingent debt instruments to the forefront of policy debates as a strategy to avoid costly defaults (United Nations, 2006; Blanchard et al., 2016; IMF, 2017; IMF, 2020). There is also a substantial theoretical literature focusing on the merits of indexing sovereign debt to real variables to help with macroeconomic stabilization and risk sharing. Shiller (1993), Hatchondo and Martinez (2012), Benford, Ostry, and Shiller (2018) and Kim and Ostry (2021) argue that GDP-indexed bonds could allow governments to reduce both the cyclicality of fiscal policy and default risk while improving risk sharing with international creditors. More generally, the benefits of improving international risk sharing have been discussed extensively since the seminal work by Backus et al. (1992). Several recent studies focusing on the advantages of fiscal unions have found that the gains from improving regional stabilization and risk sharing are quantitatively important (Beraja, 2023; Farhi and Werning, 2017). Obstfeld and Peri (1998) argue that state-contingent debt could replicate these features without having to resort to a politically unfeasible combination of taxes and transfers.[1]

Despite these well-understood advantages, the use of state-contingent debt instruments is scarce in practice and countries have not been able to issue such financial instruments at a reasonable premium—as in the recent cases of Argentina (2005), Greece (2012) and Ukraine (2015). Surprisingly, while some practical implementation challenges have been discussed among policy makers, there is little theoretical analysis investigating them and the lack of indexation in sovereign debt markets remains a puzzle. IMF (2017) and Benford et al. (2018) point to myopia on the part of issuers, who might be out of office before the gains fully materialize.[2] Krugman (1988) argues that GDP-indexed bonds could create moral hazard problems by deincentivizing

[1] Beraja (2023) and Farhi and Werning (2017) show that the efficient risk sharing arrangement within a fiscal union could be achieved through a simple contingent transfer rule that bear resemblances to a GDP-indexed bond.

[2] Amador (2012) and Aguiar et al. (2020) study sovereign debt models including political myopia that could arise because of political polarization or political turnover.

the government to conduct growth-friendly policies or misreport GDP statistics. For instance, Morelli and Moretti (2023) discuss that Argentina reported lower than actual inflation to reduce the real value of payments on inflation linked debt. However, this misreporting of inflation raised reported real growth and, in turn, increased the payout on GDP-linked debt. Moreover, many countries have issued inflation-indexed bonds with success. Thus, these arguments do not seem to be empirically relevant.[3] Others argue that markets for these instruments tend to be shallow and, thus, these bonds would carry a large liquidity premium. Moretti (2020) investigates this liquidity channel and finds that state-contingent debt is still welfare-improving. Overall, there are no compelling arguments in the literature to outweigh the aforementioned merits of indexation and justify their little use in practice.

This paper aims to fill this gap and proposes a novel mechanism to understand why state-contingent debt has only been issued on a modest scale and severely underpriced. We evaluate state-contingent instruments in light of a sovereign default framework à la Eaton and Gersovitz (1981) with long-term debt, augmented with international lenders who fear model misspecification. In our environment, foreign lenders have doubts about the probability model of the small open economy's exogenous income process and guard themselves against this ambiguity by forming pessimistic expectations. Lenders with preferences for robustness distort probabilities about exogenous shocks in an endogenous way by boosting the probability of low-utility states and seek decision rules that perform well under these worst-case distributions. In the model, low utility events are associated with periods of high default risk and, in the case of state-contingent debt, periods in which stipulated repayments are low.

The presence of robust lenders creates a tradeoff for the optimal design of state-contingent debt. Obtaining more insurance from lenders increases the relevance of their doubts about the true stochastic process governing the indexing variable. On the other hand, contingency in stipulated repayments can help reduce the probability of default and the volatility of consumption

[3]Moreover, the moral hazard problem could also apply to non-contingent debt if the costs of defaults vary with income as is often assumed in the sovereign default literature.

while improving risk sharing with the foreign creditors as in previous studies (e.g., Hatchondo and Martinez, 2012). The typical design of state-contingent instruments that countries have recently used involves a threshold below which no payments are made (Argentina 2005, Greece 2012, and Ukraine 2015 are 21st century examples). This threshold structure, especially when the threshold is imposed at relatively high levels of income, is sensitive to the types of probability distortions that robust lenders worry about, which leads them to heavily discount the bond. Threshold indexation also provides little insurance in exchange for the repayment variance it imposes on lenders. In line with the empirical evidence, our model with robust lenders generates wide spreads for this threshold-type of instruments, which also lead to equilibrium welfare losses compared to noncontingent debt and, thus, explains why governments seem so reluctant to issue these instruments.

Robustness is a standard device in the asset-pricing literature which enables more realistic market prices of risk. In the context of noncontingent debt, Pouzo and Presno (2016) show that augmenting the baseline sovereign default model with robust lenders is essential to simultaneously match the spread dynamics and the frequency of default observed in the data. They also show that the same model with lenders with standard CRRA preferences and no robustness would generate counterfactually high bond prices for plausible levels of risk aversion. Similarly to the equity premium puzzle, for large values of risk aversion the model without robustness can generate high spreads for noncontingent debt at the expense of an extremely low risk-free rate at odds with the data.

We motivate this framework as a way to capture concerns that the model used to fit the past GDP series, or the underlying variable associated with the state-contingent bond, does not (yet) accurately capture all relevant features of the economy. While we do not pursue them, there are other reasons why the robust-lenders model can be appealing. One is that robustness captures potential concerns that the evolution of the country's GDP may be different in the future than the current estimate of what it has been in the past. It could be that, as a result of (the success of) moving to state-contingent debt, the government modifies its fiscal or monetary policies. It could even be that reported GDP and actual GDP diverge because of incentives to misreport statistics. Finally, it could be that issuing state-contingent debt signals some underlying type of the government: maybe an impatient one who seeks to finance irresponsible policies, or a responsible one who seeks to improve risk-sharing with the rest of the world. While we would prefer explicit models of most of these features, robustness could in principle be interpreted as representing some of them, informally capturing the degree of credibility that lenders assign to the countries they lend to.

The sovereign default framework à la Eaton and Gersovitz (1981) is commonly used for quantitative studies of sovereign debt and has been shown to generate a plausible behavior of sovereign debt and spreads. Formally, we analyze a small open economy that receives stochastic endowments of a single tradable good. The government is benevolent, issues long-term debt in international markets, and cannot commit to repay its debt. While not in default, the government issues debt which is purchased and priced by competitive foreign lenders. Following Pouzo and Presno (2016), we extend the canonical model by assuming that these foreign lenders are endowed with multiplier preferences (Hansen and Sargent, 2001), a tractable way to introduce concerns for robustness. In our baseline model, the government issues noncontingent bonds. By varying the asset structure, we examine the equilibrium consequences of making debt payments linked to the realization of the income shock in different ways.

We structure our discussion around the GDP warrants issued by Argentina as part of its 2005 debt restructuring. Costa, Chamon, and Ricci (2008) find that these bonds traded at large premia: around 300bps which they attribute to the default risk of other securities, and a residual of between 600 and 1200bps which they interpret as a premium for 'novelty.' We first calibrate our model to match key moments in the data for Argentina assuming that the government only issues noncontingent debt. Then we evaluate the effects of indexation by assuming the government can issue a state-contingent bond (which we label as 'threshold' bond) that resembles the structure of the GDP warrant issued by Argentina. With rational-expectations lenders, this threshold bond provides welfare gains to the country relative to noncontingent debt. The threshold bond also realizes about two-thirds of the welfare gains of optimal state-contingent debt when facing rational-expectations lenders. These welfare gains, however, are overturned with robust lenders, who charge high spreads as their probability distortions magnify the likelihood of states with lower payments—an ambiguity spread. This ambiguity spread explains most of the so-called novelty premium on Argentine GDP warrants and, more broadly, the lack of appetite for issuing these instruments.

We then characterize the structure of the optimal state-contingent bond and show how it is affected by the degree of robustness. In contrast to the commonly used threshold bond, the optimal design generates substantial welfare gains, although these gains are decreasing in the level of robustness. For tractability, we first characterize the optimal design of state-contingent debt in a stylized version of our model. We show how mean-preserving differences in the structure of promised repayments –which have no impact on rational-expectations, risk-neutral lenders– may imply large differences in the spreads charged by lenders who fear model misspecification. The lenders' robustness limits the scope for risk-sharing in particular ways: the optimal debt structure features less contingency, lower slopes, and an avoidance of regions with zero or low stipulated repayments. For a given level of insurance, the government would like to minimize the contingency in stipulated repayments in order to prevent costly probability distortions. But the degree of hedging it can attain is itself limited by default risk ex-post. As the insurance-cost schedule moves out with robustness, welfare gains from optimal state-contingent are decreasing in the degree of robustness. These insights are preserved in the quantitative version of our model, where for computational reasons we only optimize over a parametric family of state-contingent instruments.

Overall, our findings cast doubts about the desirability of using the threshold type of state-contingent bonds that countries have been issuing in the past and demonstrate how the optimal bond indexation depends on the degree of lenders' preferences for robustness. Our model rationalizes the so-called novelty premia in threshold bonds as ambiguity premia associated with the type of contingency these bonds introduce. These premia overturn the well-understood advantages of indexed bonds (which are present in our model) and create substantial welfare losses for the government. Robustness can therefore explain why state-contingent bonds with GDP thresholds for repayment have not been generally regarded as successful by their issuing countries.

Discussion of the Literature Our analysis builds on and extends three branches of the literature: sovereign default, robust control theory, and the implications of state-contingent debt. First, our study is related to the recent literature on quantitative models of sovereign default that extended the approach developed by Eaton and Gersovitz (1981), starting with Aguiar and Gopinath (2006) and Arellano (2008). Different aspects of sovereign debt dynamics and default have been analyzed in these quantitative studies. Excellent surveys of the literature on sustainable public debt and sovereign default can be found in handbook chapters by Aguiar and Amador (2014), Aguiar, Chatterjee, Cole, and Stangebye (2016), D'Erasmo, Mendoza, and Zhang (2016), and Martinez, Roch, Roldán, and Zettelmeyer (2023).

Our study also relates to the literature on robust control methods pioneered by Hansen and Sargent (2001, 2016). A growing theoretical macro literature extends canonical models to the case in which the social planner and/or private agents fear model misspecification and search for robust policies under worst-case scenarios (Adam and Woodford, 2012; Ferriere and Karantounias, 2019; Bennett, Montamat, and Roch, 2023). We relate closely to Pouzo and Presno (2016) who study a sovereign default model with robust international lenders in the context of noncontingent debt. Their analysis demonstrates how the introduction of robust lenders improves the quantitative performance of sovereign default models. Robustness helps match bond spreads dynamics observed in the data without resorting to counterfactually high default frequency by historical standards. We then study how international investors' concerns about model misspecification affect the spreads, welfare implications, and optimal design of state-contingent bonds.

Finally, our paper naturally relates to a literature concerned with the implications of state-contingent debt. Borensztein and Mauro (2004) focus on the implications and benefits of state-contingent debt for the cyclicality of fiscal policy. Durdu (2009) shows that the degree of indexation should be optimally chosen to smooth sudden stops, and that this optimal degree of indexation depends on the persistence and volatility of the shocks an economy faces. More closely related to our paper, Hatchondo and Martinez (2012) and Bertinatto et al. (2017) study the effects of introducing income-indexed bonds into standard quantitative sovereign default models. Both studies find that, in models without robustness, GDP-indexed securities support large welfare gains when designed optimally. These papers emphasize that GDP-linked bonds allow the government to eliminate default risk while increasing indebtedness, thereby reducing the equilibrium volatility of consumption relative to income. However, their baseline model with one-period bonds generates counterfactual bond spread dynamics and debt levels. For example, the benchmark calibration in Hatchondo and Martinez (2012) generates a 3% mean spread and 4% debt-to-income ratio in their simulations. We build on these papers by clarifying how robustness on the part of lenders helps match empirical (low) bond prices which ultimately overturns the conclusions on welfare implications of state-contingent debt for the designs observed in reality.

Layout The remainder of the paper is structured as follows. First, section 2 documents recent country experiences with sovereign state-contingent bonds and motivates the mechanism proposed in the paper. Section 3 lays out a simple two-period model that illustrates how the optimal design of state-contingent bonds and the associated welfare implications depend on the lenders' preferences for robustness. Section 4 introduces the quantitative model. Section 5 contains our main results on the equilibrium effects of different state-contingent debt instruments. Finally, section 6 concludes.

2 Recent experiences with state-contingent debt and the case for robustness

State-contingent debt instruments are not used frequently in sovereign borrowing. Subsection 2.1 summarizes some recent cases, along with a description of the contingency involved in the various issuances, and shows that these bonds have traded at a large premia. Subsection 2.2 discusses why we consider that robustness provides a better rationale for this underpricing over alternative explanations in the literature.

2.1 The design of state-contingent debt and the 'novelty' premium

Pina (2022) compiles 38 instances of issuances of sovereign state-contingent debt, ranging from cotton bonds issued by the Confederate States of America in 1863 to the IBRD Cat bonds issued in 2018 by Peru, Colombia, Chile, and Mexico to stipulate lower debt payments in case of earthquakes. The vast majority of bonds are structured in a way that promises reduced or no payments if some measure of income (such as output or a key export price, among others) falls below a certain threshold.

Costa, Chamon, and Ricci (2008) describe in detail the GDP-warrants issued by Argentina as part of its 2005 debt restructuring. This contract was characterized by a threshold and a slope. Payments only occured if the state of the economy satisfied three conditions. First, real GDP growth must exceed 'baseline' GDP growth in the reference year.[4] Second, the level of real GDP had to be higher than the compounding of the baseline growth rates. Finally, payments only occured if the cumulative amount of past payments was below another threshold (of 48 cents of the currency of denomination per unit of security). If the threshold was satisfied, the slope component of the bond meant that payments were a fraction of the difference between the actual and baseline levels of real GDP. Costa, Chamon, and Ricci (2008) use Monte Carlo simulations based on historical data to compute theoretical prices for the GDP warrants from 2005 to 2007. They

[4]Baseline GDP growth was set by the authorities to gradually converge from an initial level of 4.3% in 2005 to a long-run level of 3% at the maturity of the bond in 2034.

find wide spreads: between 300 and 400bps which they attribute to the default risk of other securities, and a residual of 1200bps (which declined over time to about 600bps) which they interpret as a premium for 'novelty.'

The decline in the residual premium featured in Costa, Chamon, and Ricci (2008) could indeed justify labeling it as 'novelty' implying that these were relatively exotic assets and, thus, market participants needed time become familiar with the trading of these state-contingent bonds. Moreover, the severe underpricing could also be related to a poor economic outlook specific to Argentina, given its fragile macroeconomic framework. However, Igan et al. (2021) update the exercise in Costa, Chamon, and Ricci (2008) with more recent data (also extending to additional countries), and their results suggest otherwise. Figure 1 shows the estimated spread decomposition for the GDP-warrant issued by Argentina, Greece and Ukraine. The residual premium, which they label as 'SCDI Premium,' has remained at a high level in the case of Argentina over a period of 15 years. They also find a significant residual premium in the recent issuances by Greece and Ukraine. These results indicate that the underpricing of these instruments is not a country-specific issue and does not seem to be related to the novelty of the instrument.

In contrast, many countries implicitly index their debt portfolio through a combination of local-currency, foreign-currency, and inflation-indexed debt, which builds in state-contingency but avoids thresholds. Low spreads in such cases is consistent with our model, where linear indexation is generally superior to threshold indexation. The main reason behind the issuance of threshold bonds for GDP indexation is that GDP-linked bonds have been typically issued in the context of debt restructurings. The threshold design was typically the outcome of the desire to bridge the gaps in views regarding economic outlook and debt servicing capacity between creditors and debtors. The underlying idea was that this bond structure may allow for appropriately conservative base case scenarios that minimize the risk of future defaults. The main message of our paper is precisely that the lack of success of GDP-linked bonds is due to threshold indexation and robust lenders.

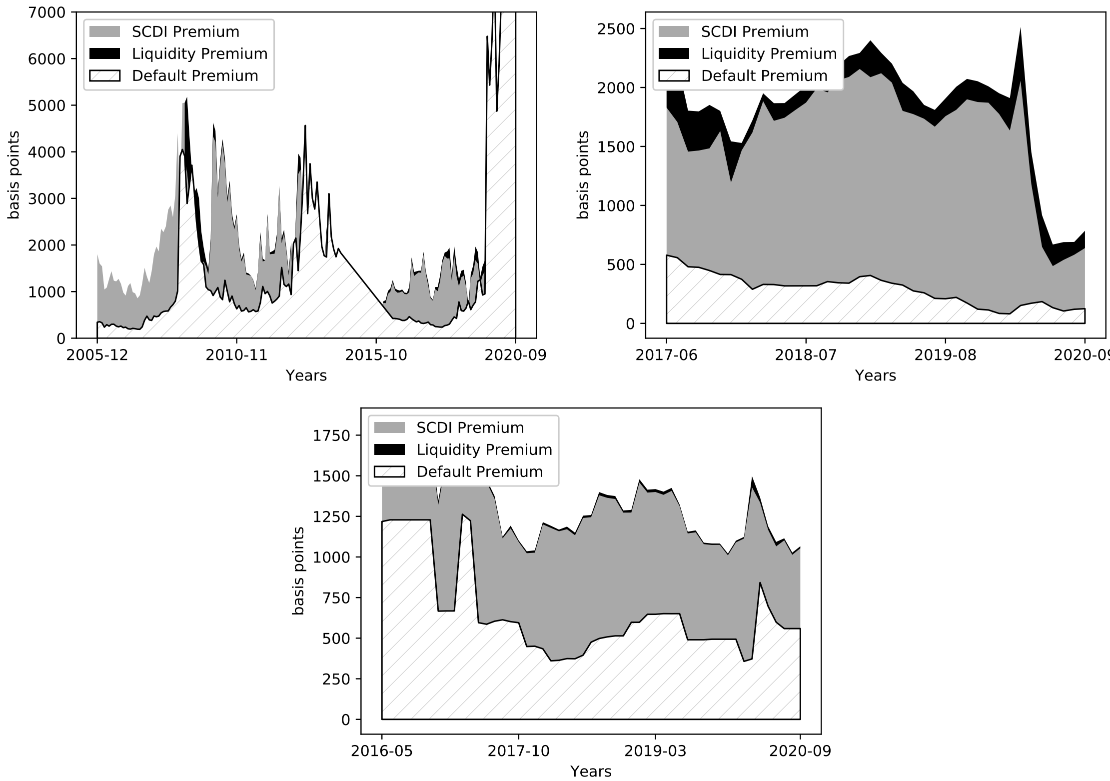

Figure 1: GDP-linked security premia.
Note: The figure shows the estimated spread decomposition in Igan et al. (2021) for the GDP-warrants issued by Argentina (top left), Greece (top right) and Ukraine (bottom).

2.2 Why robustness?

Among the existing arguments for the lack of indexation in sovereign debt markets discussed in the Introduction, concerns about data accuracy and moral hazard related to misreporting statistics seem to be the most shared in policy circles. However, these concerns should not be overemphasized. First, these arguments should also apply to inflation-linked bonds which many countries have managed to trade a reasonable prices.[5] Second, in the case of GDP-linked bonds, governments would pay significant political costs for underreporting growth. Moreover, long maturities could correct for the moral hazard problem, as the GDP growth rate would be less manipulable over the longer term. In fact, in the case of Argentina, the government has been criticized for allegedly *over*reporting growth and underreporting inflation. In the case of Greece, the issuance of

[5] As discussed above, inflation-linked bonds typically do not involve repayment thresholds.

GDP-linked securities occurred under the scrutiny of the European Commission, the ECB and the IMF. The oversight of these international organizations reduces the margin for data inaccuracies or misreporting, but nevertheless the Greek GDP-linked warrants were also heavily discounted (see Figure 1).

The theoretical values for the Argentina GDP-warrant estimated in Costa, Chamon, and Ricci (2008) were significantly above those in market reports. This valuation discrepancy was mainly related to the assumptions regarding the growth process. While their calculations were based on the average of the Consensus Forecast, Costa, Chamon, and Ricci (2008) also state that investment banks' pricing models exhibited more conservative growth forecasts. This suggests that investors' forecasts of GDP growth were in general pessimistic and underpredicted Argentina's economic recovery. Finally, Cantor and Packer (1996) argues that financial markets regard sovereign ratings with skepticism, and Grosse Steffen and Podstawski (2017) provide some more recent evidence of investor pessimism and ambiguity-aversion in sovereign debt markets, in line with Pouzo and Presno (2016).

Our framework with robustness captures investors' pessimism when pricing sovereign debt. The discrepancy in growth forecasts reflects that investors have not settled on a 'true' model governing the data and/or are concerned about specification errors behind the processes used to model economic variables.[6] Thus, robust lenders charge an uncertainty premium by slanting probabilities towards worse but plausible scenarios to guard themselves against possible errors in the estimated process.

3 A stylized model of sovereign default with robustness

This section presents a stylized sovereign default model to conceptually explore the interaction between state-contingent debt and robustness. A small open economy, populated by a govern-

[6] As mentioned before, our framework could also be reinterpreted to accommodate concerns about data reliability and misreporting. Actual data could be conceptualized as being generated by a perturbation of the statistical process describing reported data. External scrutiny and other forces discussed above would limit the size of this perturbation, which in our framework maps into a cap on the contribution of these sources to the overall robustness parameter.

ment and a representative agent, faces risk-neutral competitive foreign lenders. The world lasts for two periods in which the government receives endowments $(y_1, y_2) > 0$. There is uncertainty about z which determines the value of income in the second period $y_2(z)$.

Assets Only one type of security is traded.[7] When the government issues debt, it promises a repayment $R(z)$ in state z of the second period. Different specifications of the stipulated repayment function R represent different types of state-contingent debt structures. We focus on four different types of repayment functions summarized in Table 1.

Table 1: Stipulated repayment functions

Type of debt	**Stipulated repayment**		
Noncontingent debt	$R(z)$	$=$	1
Linear indexing	$R^{\alpha}(z)$	$=$	$1 + \alpha\left(y_2(z) - \mathbb{E}_1\left[y_2(z)\right]\right)$
Threshold debt	$R^{\tau}(z)$	$=$	$\mathbb{1}\left(y_2(z) > \tau\right)$
Optimal design	$R^{\star}(z; \theta)$		chosen state by state

Noncontingent debt promises a constant repayment regardless of the state, while the repayment of linearly-indexed debt depends on the difference between realized output and its mean, with a slope parameter of $\alpha \geq 0$.[8] Threshold debt pays only if the state is above a minimum level. Finally, we compute the debt structure that maximizes the utility of the government by promising non-negative repayments $R^{\star}$ state-by-state in a flexible manner. Our notation anticipates that the optimal design depends on the lenders' preferences as summarized by the robustness parameter θ introduced below.

Government The government is benevolent and makes decisions on a sequential basis. The government acting in period $j \in \{1, 2\}$ maximizes $\mathbb{E}\left[\sum_{t=j}^{2} \beta_b^{t-j} u\left(c_t\right)\right]$, where $\mathbb{E}$ denotes the expectation operator, $\beta_b \in (0, 1]$ is the government's discount factor, c_t represents period-t con-

[7] In line with previous studies (Borensztein and Mauro, 2004; Durdu, 2009; Hatchondo and Martinez, 2012; Hatchondo et al., 2016; Bertinatto et al., 2017; Kim and Ostry, 2021; Sosa-Padilla and Sturzenegger, 2021), we abstract from the portfolio problem and compare the cases of noncontingent and contingent debt.

[8] In the model, we use $\max\{0, R^{\alpha}(z)\}$ when considering linearly-indexed debt to keep repayments nonnegative. We skip the max in the description to keep the notation succint.

sumption in the economy, and the utility function u is increasing and concave. The government borrows to finance consumption in period 1, taking as given the stipulated repayment function R. The government may choose to default in period 2, in which case it does not pay the debt but loses $\phi(z)$ of the endowment y_2. We consider a standard quadratic specification for the output-cost function, meant to make the cost of default increasing and convex in output

$$\phi(z) = d_1 y_2(z)^2$$

The government understands the pricing function $q(b)$ that foreign lenders offer for an issuance level b. For ease of notation, we omit the dependence of q on R and θ. The government's problem is to choose debt and consumption to solve

$$\begin{aligned} V(\theta, R) = & \max_b u(c_1^b) + \beta_b \mathbb{E}\left[u(c_2^b)\right] \\ \text{subject to } \; & c_1^b = y_1 + q(b)b \\ & c_2^b = y_2(z) - \phi(z)d(b,z) - (1 - d(b,z))R(z)b \end{aligned}$$

where $d(b,z)$ takes the value of 1 if the goverment defaults in state (b,z) and 0 otherwise. $V(\theta, R)$ denotes the equilibrium value attained by the government when it faces lenders with robustness θ and issues debt with stipulated repayment R. It is common knowledge that default takes place if and only if $u\left(y_2(z) - \phi(z)\right) > u\left(y_2(z) - R(z)b\right)$.

Lenders We focus on the interaction between the design of the debt instrument and the lenders' degree of robustness. Following Hansen and Sargent (2001) and Pouzo and Presno (2016), we assume that foreign lenders feature *multiplier preferences* to capture concerns about potential model misspecification. Multiplier preferences lead our lenders to price assets by distorting their approximating or benchmark model.[9]

[9] Pouzo and Presno (2016) provide a thorough discussion of robustness in the context of sovereign debt models.

Standard arguments from the robustness literature allow us to write the lenders' problem as[10]

$$
\begin{aligned}
&\max u(c_1^L) - \frac{\beta}{\theta} \log\left(\mathbb{E}\left[\exp(-\theta v_2^L)\right]\right) \\
\text{subject to } \; & v_2^L = u(c_2^L) \\
& c_2^L = w_2 + (1 - d(b,z))R(z)b \\
& c_1^L = w_1 - q(b)b
\end{aligned}
$$

where (w_1, w_2) are the lenders' endowments in periods 1 and 2, respectively.[11]

The lenders' first-order conditions yield a pricing equation for the debt

$$
u'(c_1^L)q(b) = \beta\mathbb{E}\left[\frac{\exp(-\theta u(c_2^L))}{\mathbb{E}\left[\exp(-\theta u(c_2^L))\right]} u'(c_2^L)(1 - d(b,z))R(z)\right]
$$

where $M = \beta\frac{\exp(-\theta u(c_2^L))}{\mathbb{E}\left[\exp(-\theta u(c_2^L))\right]}$ augments the stochastic discount factor. The parameter θ controls the degree of ambiguity aversion. This Euler equation makes it clear that the model converges back to expected utility with rational expectations as $\theta \to 0$. In our baseline, lenders have per-period payoff linear in consumption, while also being uncertainty averse or ambiguity averse.[12]

Ambiguity premia The robust-lenders model links bond prices and spreads to features of equilibrium expectations about debt repayment. For risk-neutral (but still robust) lenders, we have

$$
\begin{aligned}
q(b; R, \theta) &= \beta\mathbb{E}\left[\frac{\exp(-\theta c_2^L)}{\mathbb{E}\left[\exp(-\theta c_2^L)\right]}(1 - d(b,z))R(z)\right] \\
&= \underbrace{\beta\mathbb{E}\left[(1-d)R\right]}_{=q_{RE}} + \underbrace{(1 - \mathbb{P}(d))\operatorname{cov}(M, R)}_{=q_\theta^{cont}} - \underbrace{\mathbb{E}\left[R\right]\operatorname{cov}(M, d)}_{=-q_\theta^{def}} \qquad (1)
\end{aligned}
$$

[10] See Appendix A.

[11] If lenders are risk-neutral, the relative size of their endowment (w_1, w_2) is irrelevant for outcomes (see Pouzo and Presno, 2016, for a proof in the context of noncontingent debt). In the case of risk-averse lenders, the relative size of their endowments can also be important in shaping their risk-appetite. Moreover, in general, lenders can be affected by developments in the economy through a correlation between these quantities and the endowment shocks.

[12] We leave the lenders' utility function general, even though we focus on the risk-neutral case. In general, it can be jointly calibrated along with the robustness parameter θ to match asset-pricing evidence. Another alternative is to calibrate θ to a reasonable model error-detection probability.

Equation (1) breaks up the debt price into a rational-expectations component q_{RE} and two components that depend on the degree of robustness. The first of them, q_θ^{cont}, reflects ambiguity in the contingency of the debt contract itself: given the repayment probability, it is proportional to the covariance between the stochastic discount factor M and the stipulated repayment R. The second one, q_θ^{def}, reflects ambiguity in the default strategy: controlling for the average level of stipulated payments, it is proportional to the covariance between the stochastic discount factor and the repayment strategy. Because the lenders' marginal utility is decreasing in debt payments, these covariances will be respectively negative and positive. Both ambiguity terms contribute to lower bond prices and larger spreads.

We compute and decompose spreads as follows. Let $r = \frac{\mathbb{E}[R]}{q}$ be the implicit interest rate and $r - r^\star$ be the spread, where $r^\star = 1/\beta - 1$ is the international risk-free rate. We define the rational-expectations spread as $spr_{\mathrm{RE}} = \frac{\mathbb{E}[R]}{q_{\mathrm{RE}}} - r^\star$, the premium from the ambiguity of contingent repayment as $spr_\theta^{cont} = \frac{\mathbb{E}[R]}{q_{\mathrm{RE}}+q_\theta^{cont}} - \frac{\mathbb{E}[R]}{q_{\mathrm{RE}}}$, and the premium from the ambiguity of default as $spr_\theta^{def} = \frac{\mathbb{E}[R]}{q_{\mathrm{RE}}+q_\theta^{cont}+q_\theta^{def}} - \frac{\mathbb{E}[R]}{q_{\mathrm{RE}}+q_\theta^{cont}}$.

The robust-lenders model allows us to characterize the probability distortions that underpin debt prices in an equilibrium. Let the *distorted expectation* of a random variable X be the objective expectation of the product of X with a likelihood ratio

$$\tilde{\mathbb{E}}\left[X\right] = \mathbb{E}\left[\frac{\exp(-\theta u(c_2^L))}{\mathbb{E}\left[\exp(-\theta u(c_2^L))\right]}X\right] \tag{2}$$

As compared to the expectation taken with the objective probability measure, the distorted expectation magnifies the likelihood of states for which the lenders' utility is low. Different designs for government debt (different R functions) lead to different equilibrium outcomes for lenders, which in turn support different worst-case models and different probability distortions.

3.1 Probability Distortions

To investigate the effect of robustness on state-contingent debt prices, we solve the stylized model for different repayment functions R and different levels of the robustness parameter θ. We leverage Equation (2) to recover the probability distortions used by lenders to evaluate debt payoffs in each equilibrium.

Table 2 summarizes our parametrization. We keep close to the Argentinian GDP-linked bonds described in Costa, Chamon, and Ricci (2008). The parametrization is purposely artificial to highlight how the lenders' robustness interacts with the design of bonds to create spreads. In particular, we allow the robustness parameter θ to vary between 0 (rational expectations) and 4, an arbitrary level that is high enough to illustrate the forces at play. Section 4 focuses on an infinite-horizon version of the model calibrated to Argentina in which θ is controlled by model detection error probabilities.

Table 2: Parametrization of stylized model

Parameter	**Target**	**Value**
β_b	Borrower's discount rate	6% ann.
β	Risk-free rate	3% ann.
γ	Borrower's risk aversion	2
d_1	Output cost of default	20%
g	Expected growth rate	8% ann.
τ	Threshold for repayment	1
σ_z	Std. deviation of log output	0.15

One period is five years. We set the first period endowment y_1 to make $\mathbb{E}[y_2(z)] = 1 = y_1(1+g)^5$, so that g is the expected growth rate.[13] The output cost of default d_1 as well as expected growth g are set to high values to simultaneously generate high levels of debt and a low default probability, which is complicated in this stylized model by the use of one-period bonds (this difficulty is absent in our quantitative version with long-term debt). Output in the second period $y_2(z) = \exp(\sigma_z z)$ where $z \sim \mathcal{N}(0, 1)$. Other parameters are set to standard values in the literature.

[13] While foreign lenders agree with the second equality, their worst-case model will in general yield a distorted expectation $\tilde{\mathbb{E}}[y_2(z)] < 1$.

We parametrize our threshold bond structure to closely resemble the actual bond that countries have used in practice. In particular, we set the repayment threshold τ at the mean of second-period output. This is meant to replicate the fact that the Argentinian GDP-linked bond was designed to pay if output growth was above average. At the time of issuance, the Consensus Forecast for Argentina's GDP growth was about 3% over the medium-term, which coincides with the bond's main condition for repayment.

Simple state-contingent instruments We begin by analyzing equilibrium outcomes associated with noncontingent debt and threshold debt.

Figure 2 shows the probability distortions when the government issues noncontingent bonds. For ease of exposition, we fix the amount of debt issued at the optimal level when $\theta = 0$ (the rational-expectations case), so that the default probability does not vary with the degree of robustness.[14] This allows us to concentrate on the effect of robustness on prices, without the compensating reaction of the government. The top panel shows the default probability at each state as a dotted line and the distorted density (used by lenders to evaluate payoffs) in solid lines. The bottom panel shows the stipulated payment R as a dashed line and the likelihood ratios $\frac{\exp(-\theta c_2^L)}{\mathbb{E}\left[\exp(-\theta c_2^L)\right]}$ in solid lines. The distorted density used by lenders equals the likelihood ratio times the objective density.

In the case of noncontingent debt, the expected repayment is a step function of the state z: the government defaults when income is low. The lenders' stochastic discount factor is therefore also a step function of the state, as marginal utility of lenders is constant (and high) to the left of the default threshold and constant (and low) to the right of it. As the robustness parameter θ increases, robust lenders distort probabilities more by assessing the default set as more likely. For higher values of θ, therefore, the expected return of the debt (under the distorted density) is lower and lenders require higher spreads to hold it.

[14]When the government can optimize issuances as a function of θ, as we will see later, it issues less debt as lenders become more robust and charge higher spreads. This moves the default threshold to the left as θ increases.

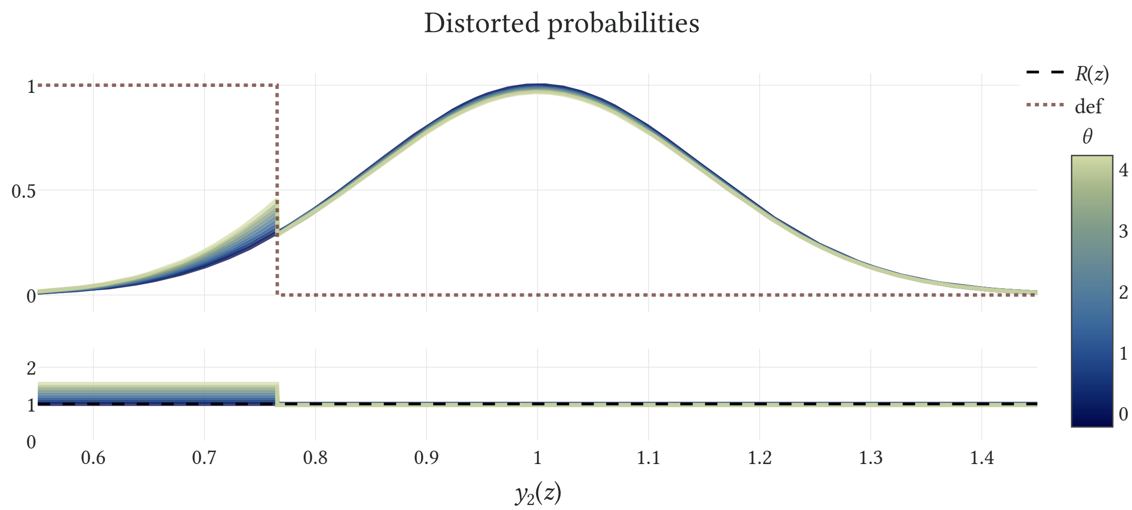

Figure 2: Distorted probabilities when the government issues noncontingent debt (amount fixed at the rational-expectations eq'm level). Top: distorted densities for each θ and default probability. Bottom: likelihood ratios (distortions) and promised repayments $R(z)$

Figure 3 considers the case of the threshold bond which promises to repay 1 unit of the good if the state is above its mean (under the approximating model which is shared by government, lenders, and Nature) and 0 otherwise. In this case, the probability distortions are much more

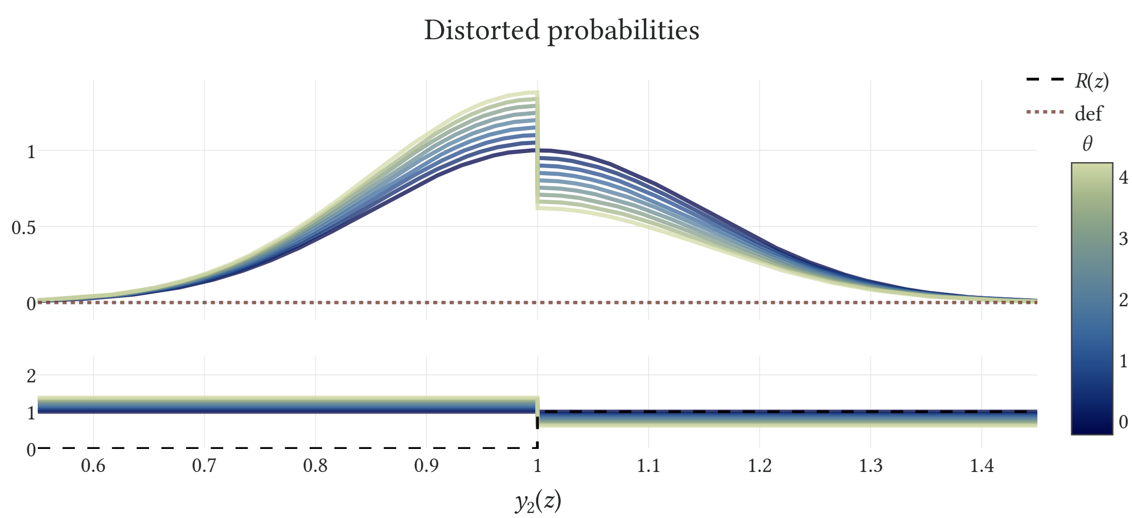

Figure 3: Distorted probabilities when the government issues the threshold bond (amount fixed at the rational-expectations eq'm level). Top: distorted densities for each θ and default probability. Bottom: likelihood ratios (distortions) and promised repayments $R(z)$

striking. Similarly to the default threshold, the jump in stipulated repayments creates a jump in the probability distortions. But the same distortion applied to an event with higher probability results in a larger change in the density. Moreover, as we will see later, the large distortions evident in Figure 3 manifest as high spreads that negate the gains from contingency in repayment.

Optimal debt design We turn our attention to the problem of how to design state-contingent debt instruments and how the optimal design changes with the degree of robustness. When facing lenders with robustness parameter θ, let $R^\star(z;\theta)$ maximize the equilibrium value attained by the government $V(\theta, R)$, subject to a non-negativity constraint.

Figure 4 shows the optimal debt design $R^\star(z;\theta) = \arg\max_R V(\theta, R)$ for each value of the robustness parameter θ, as well as the expected repayment of noncontingent debt (taking default into account). It is clear that, as θ increases, the optimal debt structure features less contingency, lower slopes, and an avoidance of regions with zero or low stipulated repayments. Figure 4

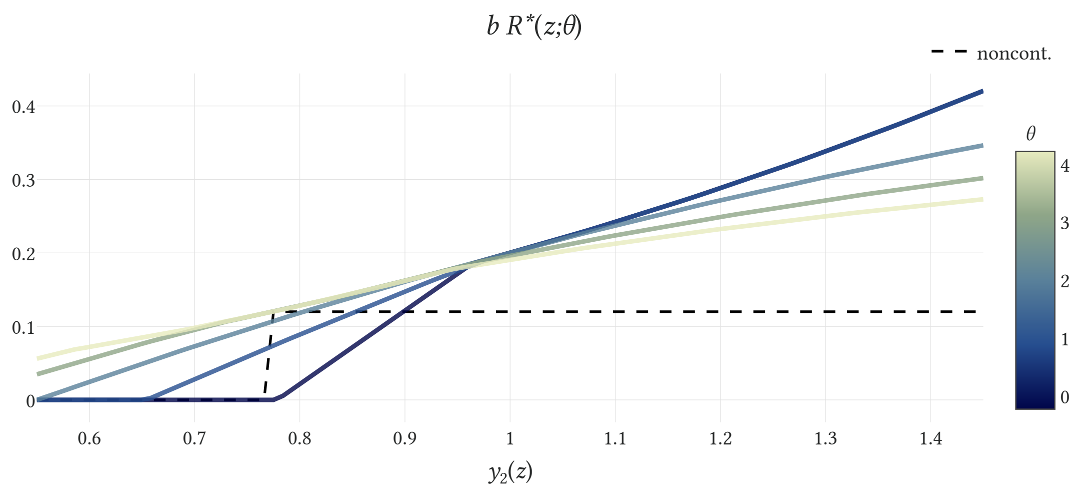

Figure 4: Optimal design of state-contingent debt for each type of lender.

sharply illustrates the tradeoffs in the debt-design problem when facing robust lenders. On the one hand, the government would like to minimize the contingency in stipulated repayments in order to prevent probability distortions. But the government also needs to minimize another source of contingency given by default risk ex-post. In low states, the government promises as much as it can credibly commit to repay.

3.2 Spreads

We now turn to how the probability distortions and concerns for model misspecification affect bond prices, issuances, and the government's welfare in equilibrium. The top panel of Figure 5 shows our decomposition of equilibrium spreads as a function of the robustness parameter θ. The bottom panel shows the issuance value $q(b)b$ as well as the welfare of the government. We measure welfare as the equivalent increase in consumption with respect to an equilibrium with the same θ but when the government issues noncontingent debt.[15]

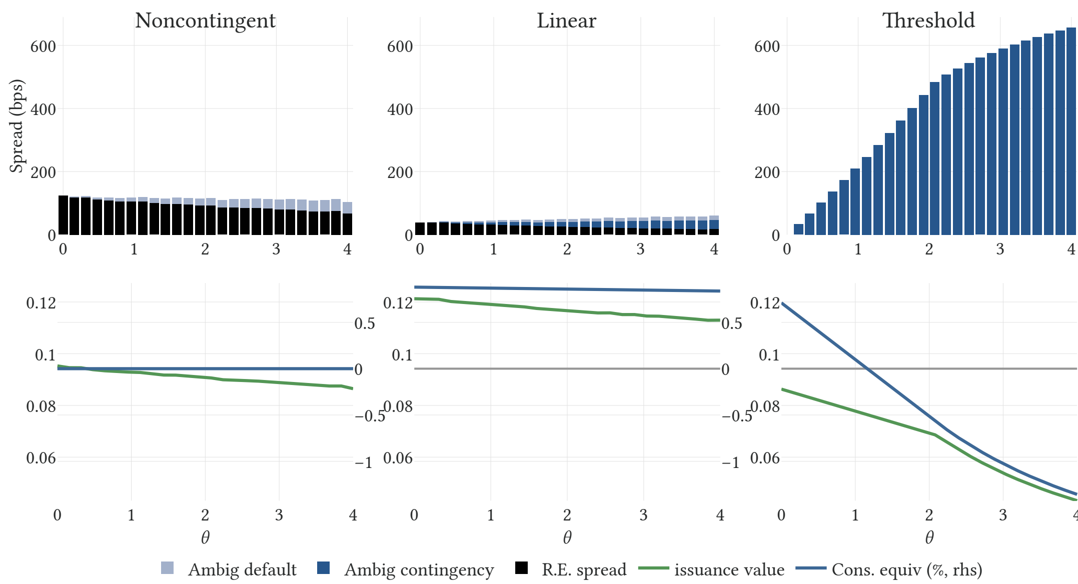

Figure 5: Spreads when the government issues the simple instruments. Bottom panel: issuance value $q(b)b$ and welfare difference from noncontingent debt

When the government issues noncontingent debt, more robust lenders charge a higher spread for the ambiguity of default. The government responds by issuing lower amounts of debt. In our parametrization, the decrease in the default probability (the amount of risk) roughly compensates

[15] Somewhat abusing notation, if

$$V(\theta, R, x) = u\left(c_1^b(1+x)\right) + \beta\mathbb{E}\left[u\left(c_2^b(1+x)\right)\right]$$

is the value attained by the government, augmenting the equilibrium level of consumption by the factor x, then in each equilibrium with bonds R and robustness θ we measure welfare by finding x to make $V(\theta, R, 0) = V(\theta, 1, x)$.

the increase in the spreads because of ambiguity (the price of risk).

Linearly-indexed debt successfully decreases the equilibrium default probability, as evidenced by lower spreads under rational-expectations. This leads to welfare gains equivalent to about 0.9% of consumption from the noncontingent debt benchmark. As robustness increases, spreads from ambiguity of contingency and from ambiguity of default open up, eroding the government's ability to issue debt and therefore welfare gains. At $\theta = 4$, however, the government still values the option to move from noncontingent to linearly-indexed debt at about 0.8% of consumption.

The picture is quite different for threshold debt. This type of debt completely eliminates default risk and therefore trades at no spreads with rational-expectations lenders. However, as θ increases, the large probability distortions discussed above support large spreads from the ambiguity of contingency. These high spreads quickly turn the welfare gains from state-contingent debt into welfare losses.

4 The quantitative model

In this section we present the infinite-horizon version of the two-period model studied in Section 3. We consider a general formulation in which the government can issue contingent defaultable debt and discuss how this nests the benchmark case with noncontingent debt as a special case.

Endowment There is a single tradable good. The economy receives a stochastic endowment stream of this good y_t. While the true stochastic process is unknown, the government trusts that the evolution of y_t is governed by the approximating model with conditional density $F(y' \mid y)$.

Government The government's objective is to maximize the present expected discounted value of utility flows of the representative household in the economy, namely

$$\mathbb{E}_t \left[\sum_{s=0}^{\infty} \beta^s u\left(c_{t+s}\right) + \epsilon_{t+s} \right],$$

where $\mathbb{E}$ denotes the expectation operator, c_t represents household's consumption, β is a discount factor, and ϵ_t is a preference shock for default or repayment.[16] The utility function is strictly increasing and concave.

As in Hatchondo and Martinez (2009), we assume that bonds promise a geometrically decreasing sequence of coupons. We expand this framework by making coupon payments vary with the realization of y_t. We allow coupon payments to depend linearly on y_t as well as threshold-based rules. In particular, a bond issued in period t promises to pay $\kappa(y_{t+1}) = \bar{\kappa}\max\{0, (1 + \alpha(y_{t+1} - 1))\mathbb{1}(y_{t+1} > \tau)\}$ units of the good in period $t + 1$ and $(1 - \delta)^{s-1}\kappa(y_{t+s})$ units in period $t + s$, with $s \geq 2$. The average coupon rate $\bar{\kappa}$ serves as a normalizing constant, as in Chatterjee and Eyigungor (2012). With parameters α, τ determining the degree of linear and threshold indexation, respectively, this bond structure allows us to keep the standard recursive formulation of the model. Note that when $\alpha = 0$ and $\tau = -\infty$ we recover the benchmark model with noncontingent bonds.[17]

At the beginning of each period in which it is not in default, the government makes two decisions. First, it decides whether to default. Second, if it chose to repay, it chooses the amount of bonds to issue (or repurchase) in the current period. We follow the literature and assume that acceleration and cross-default clauses make it impossible for the government to discriminate among its creditors or otherwise engineer partial defaults. We also assume that the recovery rate, the fraction of the loan which lenders can recover after a default, is zero.[18]

There are two sources of costs of defaulting. First, a defaulting sovereign is excluded from capital markets. In each period following a default event, the country regains access to capital markets with probability $\psi \in [0, 1]$. Second, while the country is excluded from capital markets,

[16] We follow Dvorkin et al. (2021) and introduce preference shocks for repayment and default to improve the numerical convergence of the algorithm used to solve the model.

[17] As in Section 3, we abstract from the portfolio problem and assume that only one debt instrument is traded. In line with previous studies (Borensztein and Mauro, 2004; Durdu, 2009; Hatchondo and Martinez, 2012; Hatchondo et al., 2016; Bertinatto et al., 2017; Kim and Ostry, 2021; Sosa-Padilla and Sturzenegger, 2021), we compare the cases where the government issues only either noncontingent debt or state-contingent debt.

[18] In Section 5.1 we show that our mechanism is robust to adding recovery.

it suffers an income loss of $\phi(y)$.

Lenders Our main departure from the standard setup is to allow for lenders who fear model misspecification. We follow Pouzo and Presno (2016) and assume that the lenders have per-period payoff linear in consumption, while also being ambiguity averse with respect to the probability distribution of y_t.[19] Unlike the government, lenders distrust the approximating model. They look for decision rules which are robust to possible errors in the estimated process, by slightly tilting probabilities towards worse outcomes. As before, we adopt the Hansen and Sargent (2001) multiplier preferences model, which captures ambiguity aversion with a single parameter, the inverse marginal cost of relative entropy. In this framework, model uncertainty generates a risk premium without the need for correlation between foreign investors' wealth and default. These lenders charge a premium on defaultable debt in order to guard themselves against possible specification errors in the estimated income process.

4.1 Recursive formulation

Let b denote the number of outstanding bonds at the beginning of the current period, and b' denote the number of bonds outstanding for the beginning of the following period. Let d denote the current-period default decision, with the convention that $d = 1$ when the government is in default and $d = 0$ when it is not. Let v_R and v_D denote the value functions of the government in repayment and default, respectively. Finally, let $\mathcal{V}$ denote the government's value function at the beginning of the period, before the default decision is made, which satisfies

$$\mathcal{V}(b, y) = \max\left\{v_D(y) + \epsilon_D, v_R(b, y) + \epsilon_R\right\} \tag{3}$$

where ϵ_D, ϵ_R are preference shocks for default and repayment. Both are assumed to follow *iid* type 1 extreme value distributions with scale parameter χ.[20] As shown by Chatterjee et al. (2018), this

[19] As in the stylized model, this assumption makes the size of the lenders' endowment (relative to the small open economy) irrelevant.

[20] As this shock is introduced for numerical purposes, we choose χ to be as small as possible to not affect equilibrium outcomes.

makes the net shock $\epsilon = \epsilon_D - \epsilon_R$ follow a logistic distribution and yield the following functional forms for the default probability $\mathcal{P}(b, y)$ and value function $\mathcal{V}(b, y)$

$$
\begin{aligned}
\mathcal{V}(b, y) &= \chi \log \big(\exp(v_D(y)/\chi) + \exp(v_R(b, y)/\chi) \big) \\
\mathcal{P}(b, y) &= \frac{\exp(v_D(y)/\chi)}{\exp(v_R(b, y)/\chi) + \exp(v_D(y)/\chi)}
\end{aligned}
$$

The government understands and takes as given the pricing function q used by lenders. Therefore, the conditional value functions v_R and v_D satisfy

$$
\begin{aligned}
v_R(b, y) = &\max_{b'} u(c) + \beta \int \mathcal{V}(b', y') F(dy' \mid y) \\
\text{subject to} \quad & c = y + \kappa(y) b - q(b', y)(b - (1 - \delta) b)
\end{aligned}
\tag{4}
$$

and

$$
v_D(y) = u(y - \phi(y)) + \beta \int \psi \mathcal{V}(0, y') + (1 - \psi) v_D(y') F(dy' \mid y) \tag{5}
$$

Let W denote the lenders' value function at the beginning of a period, before the default decision is made. The problem of a robust lender that fears model misspecification can be expressed in recursive form as

$$
\begin{aligned}
W(b, y) = &\max c_L - \frac{\beta_L}{\theta} \log \left(\mathbb{E} \left[\exp(-\theta W(b', y')) \right] \right) \\
\text{subject to } & c_L = \bar{z} + (1 - d(b, y)) [q(b', y)(b' - (1 - \delta) b) - b \kappa(y)],
\end{aligned}
\tag{6}
$$

where the parameter θ encapsulates the degree of lenders' ambiguity aversion, $\beta_L = 1/(1 + r)$ is their discount factor, and $\bar{z}$ is their endowment.

In this framework, bond prices are such that uncertainty-averse lenders make zero profits in expectation given their subjective beliefs. The bond price satisfies the following functional equation:

$$
q(b', y) = \int M(b', y', y)(1 - h(b', y'))(\kappa(y') + (1 - \delta) q(g(b', y'), y')) F(dy' \mid y) \tag{7}
$$

where h and g denote the future default probability and borrowing rule that lenders expect the government to follow. Expected payments from holding bonds are comprised of (potentially state-contingent) coupon payments and a resale value which summarizes the value of all future coupon payments. The lenders' stochastic discount factor, $M(b', y', y)$, is made of two parts. First, an ordinary discount factor β_L that applies in cases without uncertainty. Second, an ambiguity-aversion factor, which is given by the conditional likelihood ratio of the endogenous worst-case distorted model relative to the approximating model

$$M(b', y', y) = \beta_L \frac{\exp\left(-\theta W(b', y')\right)}{\mathbb{E}\left[\exp\left(-\theta W(b', y')\right)\right]} \tag{8}$$

Equations (3)-(7) illustrate that the government finds its optimal current default and borrowing decisions taking as given its future default and borrowing decision rules h and g. In equilibrium, the optimal default and borrowing rules that solve problems (3) and (4) must be equal to h and g for all possible values of the state variables.

Definition 1. *A Markov Perfect Equilibrium consists of a set of value functions W, $\mathcal{V}$, v_R, and v_D, a default rule h and a borrowing rule g, and a bond price function q, such that:*

(a) given h and g, $\mathcal{V}$, v_R, v_D, and W satisfy functional equations (3), (4), (5), and (6) when the government can trade bonds at q,

(b) given h and g, the bond price function q is given by equation (7), and

(c) the default rule h and borrowing rule g solve the dynamic programming problem defined by equations (3) and (4) when the government can trade bonds at q.[21]

4.2 Calibration

As in many previous quantitative studies on sovereign default (e.g., Arellano, 2008, Hatchondo and Martinez, 2009, Chatterjee and Eyigungor, 2012, Pouzo and Presno, 2016), we use Argentina

[21] Aguiar and Amador (2020) show that in an Eaton-Gersovitz model with long-term debt, there may be multiple Markov Perfect equilibria. We rule out this possibility by focusing on the equilibrium that is the limit of the equilibrium of the finite-horizon economy.

before the 2001 default as a case study. In particular, we focus on the time period starting when Argentina regained access to international capital markets (1993:I) and ending when it defaulted on its external debt (2001:IV). We solve the model numerically using value function iteration and, following Dvorkin et al. (2021), we apply dynamic discrete choice methods to overcome convergence problems that arise in models of sovereign default with long-term debt.

The utility function is assumed to display a constant coefficient of relative risk aversion denoted by γ. That is,

$$u(c) = \frac{c^{1-\gamma} - 1}{1-\gamma}.$$

The endowment y_t is autocorrelated: $\log(y_t) = \rho \log(y_{t-1}) + \epsilon_t$, with $|\rho| < 1$, and $\epsilon_t \sim \mathcal{N}\left(0, \sigma_\epsilon^2\right)$. Following Chatterjee and Eyigungor (2012), we assume a quadratic loss function for income during a default episode $\phi(y) = \max\{0, d_0 y + d_1 y^2\}$, which they show it helps the model's ability to account for the dynamics of the sovereign debt interest rate spread.

Table 3: Parameter values for the baseline parametrizations.

	Parameter	Value
Sovereign's discount factor	β	0.9504
Sovereign's risk aversion	γ	2
Preference shock scale parameter	χ	0.025
Interest rate	r	0.01
Duration of debt	δ	0.05
Average coupon rate	$\bar{\kappa}$	0.0785
Income autocorrelation coefficient	ρ	0.9484
Standard deviation of y_t	σ_ϵ	0.02
Reentry probability	ψ	0.0385
Default cost: linear	d_0	-0.2409
Default cost: quadratic	d_1	0.3018
Degree of robustness	θ	2.15
Linear coupon indexation	α	0
Repayment threshold	τ	$-\infty$

Table 3 presents our benchmark calibration. A period in the model refers to a quarter. During the time period under consideration, Argentina only issued noncontingent debt, which in our framework implies setting $\alpha = 0$ and $\tau = -\infty$. The representative household in the sovereign economy has a coefficient of relative risk aversion of 2, which is standard in studies of business cycles. The risk-free interest rate is set to 1 percent. Parameter values that govern the endowment process are chosen so as to mimic the behavior of GDP in Argentina from the first quarter of 1993 to the last quarter of 2001. The parametrization of the income process is similar to the one used in other studies that consider a longer sample period (see, for instance, Aguiar and Gopinath, 2006). We assume a probability of regaining access to capital markets that implies an average period of 6.5 years of financial exclusion, consistent with the estimates in Benjamin and Wright (2009). Following Chatterjee and Eyigungor (2012), we target an average maturity of 5 years for noncontingent bonds by setting $\delta = 0.05$.

There are four remaining parameters: the discount factor (β), the two parameters that define the output cost of defaulting (d_0, d_1), and the degree of robustness (θ). These parameter values are calibrated to match four moments in the data: (i) the level of external government debt to annual output ratio (17.4 percent); (ii) the average interest rate spread (8.15 percent); (iii) the standard deviation of the spread (4.43 percent); and, (iv) a default frequency of 3 defaults per 100 years. These targets are the same as in Chatterjee and Eyigungor (2012).[22] Argentina defaulted on its external debt five times since 1824 (see Yue, 2010), which implies an annual default frequency of 2.5%. Even though it is not clear which data values for the default frequency one should target, we choose to target this statistic because it received considerable attention in the literature. For example, Aguiar and Gopinath (2006), Arellano (2008), Hatchondo et al. (2010), Chatterjee and Eyigungor (2012), Lizarazo (2013) and Pouzo and Presno (2016) all target an annual default frequency of 3 percent. The targets for the spread distribution are taken from the spread behavior in Argentina before its 2001 default. The target for the mean debt to GDP ratio consists of the average unsecured external gross public debt between 1993 and 2001.

[22]We follow the calibration strategy of Pouzo and Presno (2016) in using θ to match the default probability.

5 Results

Table 4 reports moments in the simulations of the model for the baseline parametrization. We base statistics on pre-default simulation samples, except for the computation of default frequencies which are computed on the entire sample. We simulate the model for a number of periods that allows us to extract 1000 samples of 35 consecutive periods before a default. We focus on samples of 35 periods in order to compare the artificial data generated by the model with Argentine data from the first quarter of 1993 to the last quarter of 2001.[23] In order to facilitate the comparison of simulation results with the data, we only consider simulation sample paths for which the last default was declared at least four periods before the beginning of each sample. Default frequencies are computed using all simulation data.

Table 4 shows the simulation results of the benchmark model (only noncontingent debt) under the parametrization described above. It also shows how the simulation results change if we consider rational expectations lenders as in Chatterjee and Eyigungor (2012) (i.e., if we set $\theta = 0$). Overall, the table shows that our benchmark calibration with robust lenders is successful in accounting for the moments in the data. As in the data, in the simulations of the baseline model, consumption and income are highly correlated and the spread is countercyclical (non-targeted moments). Consumption is more volatile than income, which is consistent with the findings in Neumeyer and Perri (2005) and Aguiar and Gopinath (2007). The benchmark calibration closely matches the targeted moments as well. We note that the calibration with rational expectations lenders also approximates moments in the data reasonably well. The crucial difference is that only with robust lenders is the model able to match simultaneosuly the moments of the spread and default frequency in the data (3 percent).[24] In contrast, the model with rational expectations lenders matches the mean spread level at the expense of a much larger default probability (6

[23]The qualitative features of this data are also observed in other sample periods and in other emerging markets (see, for example, Aguiar and Gopinath, 2007, Alvarez et al., 2011, Boz et al., 2011, Neumeyer and Perri, 2005, and Uribe and Yue, 2006).

[24]Pouzo and Presno (2016) show that the model with robust lenders is also able to match other quantiles of the spread in the data.

percent).

Table 4: Data and model simulations.

	Data	Benchmark	Rational Expectations
		Targeted moments	
Spread (bps)	815	842	891
Std Spread	443	376	432
Debt-to-GDP (%)	17.4	16.7	17.1
Default Prob	3.0	3.2	6.0
		Other moments	
Std(c)/Std(y)	0.87	1.3	1.4
Corr(y,c)	0.97	0.97	0.94
Corr(y,tb/y)	-0.77	-0.6	-0.59
Corr(y,spread)	-0.72	-0.74	-0.69
DEP	-	40.1%	-

Note: Moments in the data column correspond to Argentina 1993Q1:2001Q4. Model moments are computed on pre-default samples. Spread measures are annualized. Std(X) and Corr(X, Y) refer to the standard deviation of X and the correlation between X and Y. Debt is expressed as percent of yearly GDP. The default probability is expressed in percent and computed on an unconditional simulation. DEP refers to the model detection error probability.

Table 4 reports an additional statistic which is not standard in the literature of sovereign debt: the detection error probabilities (DEP). While we calibrate the lenders' desire for robustness as discussed in the previous section, we also relate the value of θ to detection error probabilities. DEP is the probability that, given data simulated from one model, the other model's likelihood function is larger (with equal weight on which model generated the data). These are commonly used in the robust control literature to assess the plausibility of the amount of probability distortions allowed in the economy. The DEP capture the probability with which an agent, with a limited amount of data, is able to distinguish between the worst-case and the benchmark densities. If the DEP is 0, the models are so different that the agent can perfectly differentiate them. In contrast, when the DEP is 0.5 the models are identical and the agent is unable to distinguish them. A high DEP therefore suggests that the misspecification implicit in the distorted model is plausible and validates the agent desire for robustness. In the robust control literature, a calibration is deemed to imply an acceptable degree of aversion to model uncertainty when the assoaciated DEP is of at least 20% (see Barillas et al., 2009). In our case, the DEP is about 40% which implies that we allow for reasonable amounts of probability distorions in the economy.[25]

Next, we turn to the analysis of allowing the government to issue state-contingent debt. In table 5, the column "Noncontingent" shows the simulation results discussed above and the column "Threshold" shows the simulation results when the government can issue an income-indexed bond with parameters $\tau = \bar{y}$ and $\alpha = 1$. These parameter values for the state-contingent bond resemble most closely the actual bonds that countries have issued in practice. In particular, this bond structure in the model intends to capture the GDP-linked bond that Argentina issued in 2005 (as discussed in Section 3). Table 5 shows that the qualitative results from the two-period model carry over into the quantitative model. This indexed bond generates large welfare gains if the small open economy faces rational-expectations lenders due to a substantial reduction in the default probability and spreads. This allows the government to increase indebtedness and smooth consumption. The threshold bond effectively expands the government borrowing opportunities, allowing for higher indebtedness at more favorable prices. This result under rational expectations corresponds to the puzzle in the literature: the threshold-bond equilibrium generates welfare gains by improving the allocation of risk among the government and its creditors.

However, when it faces robust lenders, the government is made worse off by the introduction of the threshold bond. While the threshold bond still reduces the volatility of consumption and eliminates most of the default risk, it now leads to a large increase in bond spreads. The sovereign spread doubles from 8.4 to 16.4 percent when lenders are robust. As in Section 3, sovereign spreads not only reflect the default probability but also include amiguity premia from default and from stipulated contingency. The second row of Table 5, which shows the rational-expectations component of the spread (following the decomposition introduced in Section 3) makes it clear that

[25] We compute model detection error probabilities as the probability of misclassifying a 35-period sample (which corresponds to the length of the calibration sample) generated by the approximating model (after a 2000-period burn-in to avoid dependence on initial conditions) as coming from the worst-case model, over 2000 repetitions.

Table 5: Statistics for different debt structures.

	Rational Expectations			Benchmark ($\theta = 2.15$)		
Statistic	**Noncontingent**	**Threshold**	$\alpha = 1$	**Noncontingent**	**Threshold**	$\alpha = 1$
Spread (bps)	893	318	742	842	1636	746
o/w Spread RE	893	318	742	432	2.6	343
Spread MTI	367	318	345	947	1636	1033
Std Spread	439	133	301	376	238	282
Debt-to-GDP (%)	18.3	32.8	17.8	16.7	18.3	17.5
Std(c)/Std(y)	1.4	0.9	1.4	1.3	0.84	1.3
Corr(y,c)	0.97	0.43	0.94	0.97	0.89	0.97
Corr(y,tb/y)	-0.67	0.55	-0.51	-0.6	0.49	-0.55
Corr(y,spread)	-0.72	0.15	-0.68	-0.74	-0.55	-0.76
Default Prob (%)	6.0	1.7	5.6	3.2	0.01	2.7
Welfare Gains	-	0.94%	0.22%	-	-1.1%	0.15%
DEP	-	-	-	40.1%	31.4%	39%

Note: Threshold debt pays if income is above the mean and payments are linearly indexed with $\alpha = 1$. Spread RE refers to the rational-expectations component. Spread MTI measures the price of a marginal issuance of the 'threshold' bond in each equilibrium. Welfare gains are computed as equivalent increases in permanent consumption from zero debt, averaging over the ergodic distribution of income.

the default premium is almost non-existent, corresponding to an almost zero default probability.[26] The probability distortions imposed by robust lenders amplify the likelihood of states in which the bond gives no payments, creating large amiguity premia related to the stipulated contingency and, ultimately, welfare losses for the government. Finally, by only removing the threshold (but keeping the slope of $\alpha = 1$), a linearly indexed bond is able to produce small welfare gains, even if it only marginally reduces the default probability relative to the benchmark.

The change in bond structure affects the stochastic discount factor used by lenders and the probability distortions they use. Hence, as we move from one type of bond to another, we are not only changing the debt instrument but also potentially affecting the total amount of probability distortion allowed in the economy. If lenders have more to lose in one of the economies, they could have more pessimistic beliefs than in the other economy. This in itself could have strong implications for the pricing of the bonds. Thus, we recompute the DEP to make sure that the

[26] This decomposition also shows that in the equilibrium with noncontingent debt, approximately half of the total spread reflects ambiguity premia.

worst-case model used by lenders does not become so different from the approximating model that limited data can tell them apart. We find that DEP remain well over the 20% threshold advocated by Barillas et al. (2009) when the sovereign issues the "threshold" bond, without the need to recalibrate the benchmark value of θ.[27]

Consistent with the results in the two-period model, these findings show that, through the lens of our model, the unexplained portion of the spread of the Argentinian GDP-linked bond (which the literature has labeled as a novelty premium, see Costa, Chamon, and Ricci, 2008) is in fact an ambiguity premium. Moreover, the welfare losses associated with this bond structure could also rationalize why countries have not issued these bonds more frequently in practice.

In order to better understand the effect of robustness on bond prices, we extract the stochastic discount factor from the equilibrium with one bond and use it to price marginal issuances of another bond, rather than using the difference in equilibrium interest rates when the government reaps the full benefits of the alternative bonds and can react to the difference in pricing by reoptimizing issuances and default. That is, we ask how would an infinitesimally small quantity of bond B be priced within the equilibrium that corresponds to bond A. The row labeled Spread MTI in Table 5 shows the spreads of a marginal issuance of the threshold bond in each equilibrium. While a marginal issuance of the threshold bond trades at a low price (and broadly similar to the threshold-bond equilibrium) in the rational-expectations world, the marginal spread shoots up to almost 10 percent with robustness. Similarly, Table 6 contains the spreads of a marginal issuance of each bond we consider (including the optimal design described below), in the noncontingent bond equilibrium. Table 6 shows that for both types of lenders, small issuances of the linearly indexed bond trade at lower or almost equal spreads than the noncontingent bond. Similary, spreads are reduced even more in the case of the optimally designed bond. However, marginal issuances of the threshold bond, which trade at significantly lower spreads in the rational-expectations equilibrium, are subject to higher spreads than the noncontingent bond in the case of robust lenders.

[27] We could also recalibrate θ to ensure a constant DEP; however, since the difference in DEP is moderate, we believe that recalibrating θ would not change our main results.

Table 6: Spreads of a marginal issuance of different bonds, at the benchmark equilibrium SDF

	Noncontingent bond	Linear bond	Threshold bond	Optimal bond
Benchmark	842	845	947	829
Rational Expectations	893	849	367	634

Note: Spreads expressed in basis points and annualized.

The low marginal price of the threshold bond with robust lenders is interesting. In a two-period version of the model with noncontingent debt, the stochastic discount factor only takes two values: one for default and one for repayment. Therefore, small issuances of the threshold bond are not dramatically discounted. In the dynamic version, however, the resale price of debt varies with the state. This creates variation in the stochastic discount factor and explains the large discount applied to an ϵ-issuance of the threshold bond.

We now turn to the optimal state-contingent bond design. For each calibration, we maximize the welfare of the sovereign by choosing the parameter values α and τ. As expected, the optimal bond design depends on the type of lenders the government is facing. In particular, both the optimal threshold τ and the optimal degree of indexation α are lower when lenders feature preferences for robustness.[28] In all cases, the optimal state-contingent bond substantialy reduces both default risk and the volatility of consumption. When lenders have rational expectations, the decline in default risk implies a similar reduction in sovereign spreads. On the other hand, sovereign spreads decline much more slowly than default risk when lenders are robust, because of the ambiguity premia related to the contingency of the bond (the second row in Table 7 shows that the rational-expectations component is actually in line with the default probability). Overall, choosing the optimal state-contingent bond design results in large welfare gains, although these are larger with rational expectations lenders.[29]

We decompose the sources of welfare gains for the optimal bond following Aguiar et al. (2020). After solving the equilibrium, in simulations we compute the following auxiliary variables: con-

[28]Note that the threshold levels τ only covers the very left tail of the distribution in both cases.
[29]We also find that for a given bond structure, welfare gains are decreasing in the robustness parameter θ.

Table 7: Statistics under the optimal state-contingent bond with and without robust lenders.

	Rational Expectations		Benchmark ($\theta = 2.15$)	
Statistic	**Noncontingent**	$\alpha = 5.5, \tau = 0.904$	**Noncontingent**	$\alpha = 2.5, \tau = 0.879$
Spread (bps)	893	298	842	590
o/w Spread RE	893	298	432	205
Spread MTI	367	296	947	1098
Std Spread	439	69	376	119
Debt-to-GDP (%)	18.3	23.3	16.7	19.8
Std(c)/Std(y)	1.4	0.84	1.3	1.1
Corr(y,c)	0.97	0.93	0.97	0.97
Corr(y,tb/y)	-0.67	0.83	-0.6	-0.26
Corr(y,spread)	-0.72	-0.66	-0.74	-0.84
Default Prob (%)	6.0	2.5	3.2	1.9
Welfare Gains	-	1.6%	-	0.47%
DEP	-	-	40.1%	38.7%

Note: For each type of lender, the right column shows the optimal (α, τ) combination.

sumption without default costs c_t^R, which equals the on-path consumption when the economy is in repayment and the full level of output, without subtracting the default costs $\phi(y)$, when the economy is in default; and average consumption without default costs $\bar{c}_t^R(b_0, y_0) = \mathbb{E}\left[c_t^R \mid b_0, y_0\right]$.

Armed with these counterfactual consumption levels, we compute the counterfactual value functions without default costs and without volatility in consumption, as

$$V_{ND}(b_0, y_0) = \mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t u(c_t^R) \mid b_0, y_0\right]$$

$$V_{NV}(b_0, y_0) = \sum_{t=0}^{\infty} \beta^t u\left(\bar{c}_t^R(b_0, y_0)\right)$$

This allows us to decompose the welfare gains into gains from not suffering the output costs of default, from not suffering from volatility in consumption, and from a different average level of consumption. If $\mathcal{V}$ and $\mathcal{V}^\star$ denote the value functions from the equilibrium with noncontingent debt and the optimally designed bond, then

$$\frac{\mathcal{V}^\star(b_0, y_0)}{\mathcal{V}(b_0, y_0)} = \frac{\mathcal{V}^\star(b_0, y_0)/V_{ND}^\star(b_0, y_0)}{\mathcal{V}(b_0, y_0)/V_{ND}(b_0, y_0)} \times \frac{V_{ND}^\star(b_0, y_0)/V_{NV}^\star(b_0, y_0)}{V_{ND}(b_0, y_0)/V_{NV}(b_0, y_0)} \times \frac{V_{NV}^\star(b_0, y_0)}{V_{NV}(b_0, y_0)}$$

Table 8 presents each term of this decomposition, revealing that most of the gains from the optimal bond stem from an increase in the average consumption level.[30]

Table 8: Decomposition of welfare gains

	Total gains	From default costs	From volatility	From level
Benchmark	0.484	0.0845	0.0603	0.339
Rational Expectations	1.59	0.195	0.132	1.26

Note: Welfare gains expressed as equivalent percent increases in permanent consumption at zero debt and mean income.

5.1 Debt recovery and default probabilities

In this section, we conduct a sensitivity analysis to illustrate that the quantitative importance of the mechanism proposed in this paper is robust to the calibration of the model and allowing for positive debt recovery.

Our baseline calibration targets a default probability of 3% and assumes zero debt recovery, in line with most studies in the literature. However, 3% is a very rough estimate of the long run default probability of Argentina, because of the different ways to measure it and the peso problem.[31] If we consider only the last hundred years, the default rate for Argentina is at 5.4% (4 defaults over 73 years in good standing). Moreover, a positive debt recovery would certainly alter the quantification of the novelty premium and the role for ambiguity aversion. In particular, the disconnect between default probabilities and spreads is larger, when taking into account that mean historical mean recoveries are 60% (Cruces and Trebesch, 2013).

Thus, we recalibrate our model with noncontingent debt to match a default probability of 5.4% and capture the 40% haircut observed in the data. Following Hatchondo, Martinez, and Roch (2022), we capture in a simple fashion the positive recovery rate of debt in default observed in the data. Whenever the government declares default, coupon payments are interrupted and a haircut $\hbar$ is applied to all bonds outstanding. As before, opportunities to reaccess markets arrive

[30] The total welfare gain differs slightly from those reported in Table 7 which are averaged over the ergodic distribution of y.

[31] The fact that this data moment is difficult to pin down illustrates the difficulties faced by lenders in our model.

with constant probability ψ while the government is in default. Upon the rejection of one such opportunity, a new haircut $\hbar$ is applied to the debt, which allows the government to obtain a lower recovery rate at the expense of a longer default period. As is standard in the literature, defaulted bonds continue to be traded in secondary markets.

Table 9 reports moments computed from simulated data for this alternative version of the model. As in the main exercise, we present results for both rational expectations and robust lenders (i.e., with $\theta = 0$ but maintaining all other parameters unaltered). The table shows that the quantitative results from our benchmark calibration carry over to this alternative specification. The "threshold" bond succesfully reduces default risk and, in the case of rational-expectations lenders, also lowers spreads and generates substantial welfare gains. In contrast, robust lenders charge a meaningful ambiguity premium to this instrument (second row in Table 9). This results in welfare losses for the government. Also note that the model detection error probabilities are well above the 20% threshold as in our baseline calibration.

Table 9: Statistics based on the alternative calibration with a 40% recovery rate.

	Rational Expectations			Benchmark ($\theta = 1.946$)		
Statistic	**Noncontingent**	**Threshold**	$\alpha = 1$	**Noncontingent**	**Threshold**	$\alpha = 1$
Spread (bps)	838	209	850	764	1067	657
o/w Spread RE	838	209	850	458	0.96	341
Spread MTI	219	209	243	589	1067	679
Std Spread	744	296	535	477	199	351
Debt-to-GDP (%)	21.2	39.1	20.5	18.7	13.4	19.5
Std(c)/Std(y)	1.2	0.98	1.1	1.3	0.78	1.3
Corr(y,c)	0.95	0.54	0.94	0.97	0.94	0.97
Corr(y,tb/y)	-0.31	0.46	-0.2	-0.59	0.66	-0.56
Corr(y,spread)	-0.7	-0.31	-0.66	-0.81	-0.79	-0.81
Default Prob (%)	12.6	1.3	12.1	5.8	0.01	4.8
Welfare Gains	-	1.3%	0.19%	-	-0.81%	0.15%
DEP	-	-	-	41.6%	34.6%	39.6%

Note: Threshold debt pays if income is above the mean and payments are linearly indexed with $\alpha = 1$. Spread RE refers to the rational-expectations component. Spread MTI measures the price of a marginal issuance of the 'threshold' bond in each equilibrium.

Finally, Table 10 reports moments computed from simulated data for this alternative version of the model under the optimal state-contingent debt structure. Again, the quantitative results from our baseline calibration carry over to this alternative calibration. The optimal bond design features both a lower threshold level τ and a lower degree of indexation α when facing robust lenders.

Table 10: Statistics based on the alternative parametrization under the optimal state-contingent bond.

	Rational Expectations		Benchmark ($\theta = 1.946$)	
Statistic	**Noncontingent**	$\alpha = 5, \tau = 0.93$	**Noncontingent**	$\alpha = 2.5, \tau = 0.879$
Spread (bps)	838	130	764	376
o/w Spread RE	838	130	458	108
Spread MTI	219	160	589	718
Std Spread	744	181	477	210
Debt-to-GDP (%)	21.2	17.4	18.7	21.2
Std(c)/Std(y)	1.2	0.85	1.3	1.2
Corr(y,c)	0.95	0.9	0.97	0.86
Corr(y,tb/y)	-0.31	0.67	-0.59	-0.18
Corr(y,spread)	-0.7	-0.61	-0.81	-0.75
Default Prob (%)	12.6	1.5	5.8	1.6
Welfare Gains	-	0.92%	-	0.64%
DEP	-	-	41.6%	38.3%

Note: For each type of lender, the right column shows the optimal (α, τ) combination.

6 Conclusion

This paper studies why experiences with sovereign state-contingent debt instruments have not had the success anticipated by a literature which highlights their risk-sharing benefits. We rationalize the scarce popularity of these instruments in the context of a standard sovereign default model à la Eaton and Gersovitz (1981) with long-term debt in which foreign investors have concerns about model misspecification. International lenders are ambiguity-averse and guard themselves against possible misspecification errors in their approximating model by cautiously approaching (and pricing) the bonds offered by the issuing country. While state-contingent debt is effective in reducing default risk, robust lenders distort probabilities by assigning higher likelihood to those states where the bond promises lower repayments, resulting in an ambiguity premium associated with the contingency of the bond.

We show that this ambiguity premium can be very large when state-contingent bonds feature the threshold structure observed in recent issuances by emerging markets (e.g., Argentina, 2005; Greece, 2012; Ukraine, 2015), which results in substantial welfare losses. This bond structure, embedded in our model with robust international lenders, can account for the little use of these financial instruments and their unfavorable pricing. However, even this 'threshold' bond generates welfare gains when facing rational-expectations lenders. In this regard, we also show how the optimal bond design crucially depends on the degree of the lenders' preference for robustness.

The optimal design of state-contingent debt with robust lenders balances several forces. Lenders charge premia for ambiguity related to the stipulated payments (ex-ante contingency) as well as for ambiguity related to default (ex-post contingency). As defaulting is costly, the optimal design uses ex-ante contingency to mitigate or eliminate the probability of default ex-post. When lenders have an extreme degree of robustness, the government designs a bond that eliminates as much contingency as possible. In intermediate cases, the optimal structure enables some probability distortions in order to provide risk-sharing. The results of our calibration exercise generally support a state-contingent structure with linear indexation and potentially a threshold to cover against the extreme left tail of shocks to income.

7 Data Availability

Code replicating the tables and figures in this article can be found in Roch and Roldán (2022) in the Harvard Dataverse, https://doi.org/10.7910/DVN/MIXXXT.

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A Robust lenders' problem in the 2-period model

Foreign lenders value streams of consumption according to

$$v^L(c_1, c_2) = \min_m u(c_1) + \beta \mathbb{E}\left[m u(c_2) + \frac{1}{\theta} m \log m \right]$$
$$\text{subject to } \mathbb{E}\left[m\right] = 1$$

where m is a non-negative random variable used by the agent to twist probabilities towards worse outcomes. As is standard, we use this formulation not because we believe our agent thinks that the worst-case model does govern the state, but as a way to obtain robust policies that perform well regardless of the actual model.

We use the *multiplier preferences* formulation in which the agent entertains any probability distortion but faces a cost given by the relative entropy of the distorted distribution relative to the approximating one. The critical parameter measuring the extent of robustness is the inverse of the marginal cost of entropy for the minimizing agent, θ.

B Long-run simulations

Table 11 presents the moments of table 4 computed on the ergodic distribution instead of the pre-default samples. Results are broadly unchanged.

C Parametrization of the model with recovery

Table 11: Data and model simulations.

	Data	Benchmark	Rational Expectations
Spread (bps)	815	792	849
Std Spread	443	335	401
Debt-to-GDP (%)	17.4	13.7	12.1
Default Prob	3.0	3.2	6.0
Std(c)/Std(y)	0.87	1.1	1.2
Corr(y,c)	0.97	0.9	0.85
Corr(y,tb/y)	-0.77	-0.057	0.014
Corr(y,spread)	-0.72	-0.54	-0.42
DEP	-	40.1%	-

Note: Moments in the data column correspond to Argentina 1993Q1:2001Q4. Model moments are computed on long simulations. Spread measures are annualized. Std(X) and Corr(X, Y) refer to the standard deviation of X and the correlation between X and Y. Debt is expressed as percent of yearly GDP. The default probability is expressed in percent and computed on an unconditional simulation. DEP refers to the model detection error probability.

Table 12: Parameter values for the alternative calibration with a 40% recovery rate.

	Parameter	Benchmark Values
Sovereign's discount factor	β	0.9606
Sovereign's risk aversion	γ	2
Preference shock scale parameter	χ	0.0025
Interest rate	r	0.01
Duration of debt	δ	0.05
Average coupon rate	$\bar{\kappa}$	0.0785
Income autocorrelation coefficient	ρ	0.9484
Standard deviation of y_t	σ_ϵ	0.02
Reentry probability	ψ	0.0385
Default cost: linear	d_0	-0.2517
Default cost: quadratic	d_1	0.2952
Degree of robustness	θ	1.946
Linear coupon indexation	α	0
Repayment threshold	τ	$-\infty$
Haircut upon default	$\hbar$	0.4