Credibility Dynamics and Inflation Expectations*

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Abstract

We study the optimal design of a disinflation plan by a planner who lacks commitment and has imperfect control over inflation. The government's reputation for being committed to the plan evolves as the public compares realized inflation to the announced targets. Reputation is valuable as it helps curb inflation expectations. At the same time, plans that are more tempting to break lead to larger expected reputational losses in the ensuing equilibrium. Taking these dynamics into consideration, the government announces a plan which balances promises of low inflation with dynamic incentives that make them credible. We find that, despite the absence of inflation inertia in the private economy, a gradual disinflation is preferred even in the zero-reputation limit.

JEL Classification E52, C73

Keywords Imperfect credibility, reputation, optimal monetary policy, time inconsistency

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Introduction

Macroeconomic models give expectations about future policy a large role in the determination of current outcomes. Policy is then generally set under one of two assumptions: commitment to future actions or discretion. Attempts to model policy departing from these extreme cases have found limited success.

However, governments actively attempt to influence beliefs about future policy. Examples include forward guidance and inflation targets but also fiscal rules and the timing of introduction of policies. Such promises rarely constrain future choices, yet they can shift expectations substantially. Standard macroeconomic models cannot capture this idea directly, as expectations of the public are fully determined by the policy chosen with commitment, or with discretion as part of an equilibrium. In both cases the public understands that announcements do not bind the government in any way. Announcements do not grant any additional credibility to the policy maker, as the public is convinced of her course of action.

In this paper we develop a rational-expectations theory of government credibility and apply it to the question of optimal policy announcements. Our notion of credibility is based on the concept of reputation in game theory (Kreps and Wilson, 1982; Milgrom and Roberts, 1982). In our model the government (or central bank, or planner) could be rational and strategic, or one of many possible behavioral types described by a policy that they stubbornly follow. The public is uninformed about the government's type and makes statistical inference about it after observing the government's announcements and actions. This inference is central to our analysis because it turns out to be in the best interest of the rational type to pretend to be one of the behavioral types.

We consider a stylized environment. In the initial period, the government makes an announcement of its policy targets and is then free to choose policy. However, the private sector knows that if the government is behavioral it announced exactly what it will implement. As a consequence, the rational type has an ex-post incentive to stay close to any announced targets, which might earn it a reputation for being committed to them. The incentive exists at any positive level of reputation, though its strength depends on the announced targets. In anticipation of these interactions, the rational type chooses carefully which targets to announce. Our main question concerns the optimal policy announcement in the presence of these reputational concerns.

We set our model of reputation in a modern version of the classic environment of Barro (1986) and Backus and Driffill (1985), where a central bank sets inflation subject to an expectations-augmented Phillips curve. The monetary authority dislikes inflation but constantly faces an opportunity to engineer surprise inflation, which would deliver output closer to potential. We model these features through the standard, cashless-limit New Keynesian setup for the private economy. To focus on incentives and reputation dynamics, we abstract from an IS curve and let the government control inflation directly.

A natural definition of the government's reputation is the private sector's belief that the government is indeed the behavioral type whose plan was announced. We refer to the total, ex-ante probability of the behavioral types as the government's initial reputation. The credibility of a plan measures the proximity of expected inflation to the targets. While credibility generally increases with reputation, the insights of the reputation literature imply that our notion of credibility need not approach zero as reputation vanishes.

A key assumption we introduce is that the government exerts imperfect control over inflation, perhaps due to underlying shocks to money demand. Imperfect control masks the government's choice of policy: the private sector understands that realized inflation is only an imperfect signal of intended inflation. We consider additive and normally distributed noise which implies that the public can never be certain of the government's action. This assumption distinguishes us in technical terms from the early studies of reputation in monetary policy referenced above, where the public perfectly observes the inflation chosen by the government. But, crucially, it also creates a smooth tradeoff for the government: overshooting the target by more creates, in expectation, a larger boom accompanied by larger reputational losses.

When designing policy, the planner takes into account its own future behavior, which it can influence but not control. 'Future' governments have complete freedom and will only respect promises made at time 0 to the extent that it suits them. Preserving reputation turns out to be a powerful disciplining force for the planner's future self. Crucially, the value of reputation depends on the plan in place. Plans differ in the outcomes they intend to deliver and in how closely they are expected to be followed in the future, their credibility. Both features contribute to current outcomes through the private sector's expectations. These forces lead the planner to weigh a plan's intended outcomes against the reputation dynamics it generates.

Our main result is that the government announces a policy under which inflation starts high and diminishes gradually. Plans with gradual disinflation are more credible: having a higher target for today than tomorrow boosts the gains from sticking to the plan. This slows down the pace of reputational losses sufficiently to offset the negative effect of higher announcements on expected inflation. In contrast, the reputation literature typically considers the limit as the long-lived player becomes arbitrarily patient (Fudenberg and Levine, 1989). In that case the government can obtain a payoff arbitrarily close to its commitment payoff by announcing a static plan with zero inflation in every period.

The gradualism of our optimal policy might lead an outside observer to conclude that there is substantial inflation inertia in the economy and that the government avoids a costly recession when bringing inflation down. However, in our model past inflation does not enter the Phillips curve. Instead, gradual disinflation is a result of the dynamic incentives of the government.

A second result concerns the limit as initial reputation becomes arbitrarily small. At zero initial reputation, the only Markov equilibrium is a repetition of the static Nash equilibrium with high inflation and output at the natural level. However, as is usual in the reputation literature, even a small amount of reputation creates a large departure in behavior from the Nash equilibrium. In particular, we show that the gradualist nature of optimal announcements and the corresponding credibility dynamics are preserved at arbitrarily low levels of initial reputation. The limiting announcement, which can be interpreted as the announcement in a fully rational model where the government has mild credibility concerns, also exhibits gradualism.

Finally, even though we focus on a single long-term announcement that is naturally interpreted as a disinflation plan, our model applies more broadly to the determination and anchoring of inflation expectations. In Section 6.4 we define and employ a notion of equilibrium in which policy announcements are sequential, which leads to a similar disinflation path. Insights from related models could be relevant for bringing inflation up in a controlled manner.

1.1 *Discussion of the Literature*

We contribute to a long literature dealing with commitment, imperfect credibility, and reputation. The time-inconsistency of optimal policy (Kydland and Prescott, 1977) has long been recognized by researchers, who have set out to ask whether reputation can be a substitute for commitment.

Barro (1986) and Backus and Driffill (1985) were the first studies of monetary policy to introduce reputation via behavioral types committed to a certain policy. These and many subsequent studies (Cukierman and Liviatan, 1991; Sleet and Yeltekin, 2007; King et al., 2008; Dovis and Kirpalani, 2019) assume the government has perfect control of inflation. Thus, any deviations are detected by the private sector and fully destroy the government's reputation. In contrast, our assumption of imperfect control enables distinct tradeoffs that shape the gradualism of optimal plans.

Another line of research studies monetary policy with imperfect control by considering uncertainty about the preferences of the planner which is distinct from reputation (Cukierman and Meltzer, 1986; Faust and Svensson, 2001; Phelan, 2006). We view reputation as more directly suited to address optimal announcements, which was not the goal of the above papers.

Most closely related is the work of Lu, King, and Pastén (2016) and King and Lu (2020) who consider reputational models with imperfect control. However, their optimizing type has commitment power and the type that lacks commitment follows a fixed rule in Lu, King, and Pastén (2016), and behaves myopically in King and Lu (2020). This reversal of roles changes the underlying tradeoffs. In these papers the planner announces (and commits to) a plan that promotes separation from the alternative type. In our model, the planner directly chooses the behavioral type at the announcement stage and mimics its policy to convince the public that it is committed to it. This makes the model a natural setting for studying whether reputation-building incentives can substitute for commitment, as well as the credibility of different plans. In addition, Lu, King, and Pastén (2016) and King and Lu (2020) obtain the Ramsey plan in the limit as the planner becomes known to be the optimizing type, whereas the corresponding limiting plan in our model resembles neither commitment nor discretion.

An alternative view of reputation is given by the notion of sustainable plans (Chari and Kehoe, 1990; Phelan and Stacchetti, 2001). This literature considers subgame perfect equilibria in games between the government and the private sector applying the tools of Abreu, Pearce, and Stacchetti (1990). However, this typically generates a large set of equilibria. In fact, reputational models are often used to refine the equilibrium set. Faingold and Sannikov (2011) study a general model of reputation in continuous time which applies to our framework of monetary policy with imperfect control. They find conditions for a unique equilibrium which is Markovian in reputation, informing our restriction to Markovian equilibria. However, their model considers behavioral types with static plans, so it cannot address the gradualism of announcements we are interested in.

Even though the announcements in our model do not constrain the actions of the rational government, they are not cheap talk, as they can be sent by only one of the behavioral types. This distinguishes us from cheap talk models of monetary policy such as Stein (1989) and Turdaliev (2010).

Layout. The rest of the paper is structured as follows. Section 2 introduces our model of reputation. Notions of equilibrium are defined and discussed in Section 3. Section 4 lays out our main results and Section 5 discusses how optimal plans depend on parameters. Section 6 relates our results to other salient models. Finally, Section 7 concludes.

2. Model

We consider a government which dislikes inflation as well as deviations of output from a target $y^\star$ according to the loss function

$$L_0 = \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \left[(y^\star - y_t)^2 + \gamma \pi_t^2 \right] \right], \tag{1}$$

where y_t, π_t denote output and inflation at time t, $\gamma \geq 0$ is the relative weight on inflation, and $\beta \in (0, 1)$ is a discount factor. A Phillips curve relates current output to current and expected inflation:

$$\pi_t = \kappa y_t + \beta \mathbb{E}_t \left[\pi_{t+1} \right], \tag{2}$$

where $\mathbb{E}_t$ represents the expectations operator based on information up to time t (including π_t), and $\kappa \geq 0$ is the slope of the Phillips curve. The government has imperfect control over inflation. At time t it chooses g_t and realized inflation is

$$\pi_t = g_t + \sigma \epsilon_t, \tag{3}$$

where $\epsilon_t \overset{iid}{\sim} \mathcal{N}(0, 1)$ is realized after the government has made its choice and $\sigma > 0$. Let f_ϵ denote the density of the control shock ϵ_t.

2.1 *Announcements and behavioral types*

An announcement is a sequence $(a_t)_{t=0}^{\infty}$ of inflation targets a_t for each time t. Inflation targets are in the interval $A = [0, \pi^N]$, where $\pi^N = y^\star \frac{\kappa}{1-\beta+\kappa^2\gamma}$ is inflation in the unique Nash equilibrium of the static game. Consequently, the space of announcements is $\mathcal{A} = A^\infty$.

The government is either rational or one of many behavioral types indexed by a set $\mathcal{C} \subseteq \mathcal{A}$. A behavioral type $c \in \mathcal{C}$ is committed to making an announcement c and following it. The announcement of each behavioral type c is denoted by $(a_t^c)_{t=0}^{\infty}$ and satisfies

$$a_t^c = \chi + e^{-\omega t}(a_0 - \chi)$$

for some parameters (a_0, ω, χ). These announcements include constant, decreasing, and increasing paths for inflation, demonstrated in Figure 1. Inflation starts from a_0 and converges towards χ with a exponential decay rate of ω. When it does not lead to confusion we identify a plan $c = (a_t^c)_t$ with the triple $(a_0^c, \omega^c, \chi^c)$.

This parametrization makes $\mathcal{C}$ finite-dimensional and allows us to write each plan recursively as $a_{t+1}^c = \phi_c(a_t^c)$, where

$$\phi_c(a) = \chi + e^{-\omega}(a - \chi).$$

We assume that the government is rational with probability $1 - z$. A probability distribution on $\mathcal{C}$ with density ν describes the distribution of possible behavioral types, which have total probability z. The government's type is private information.

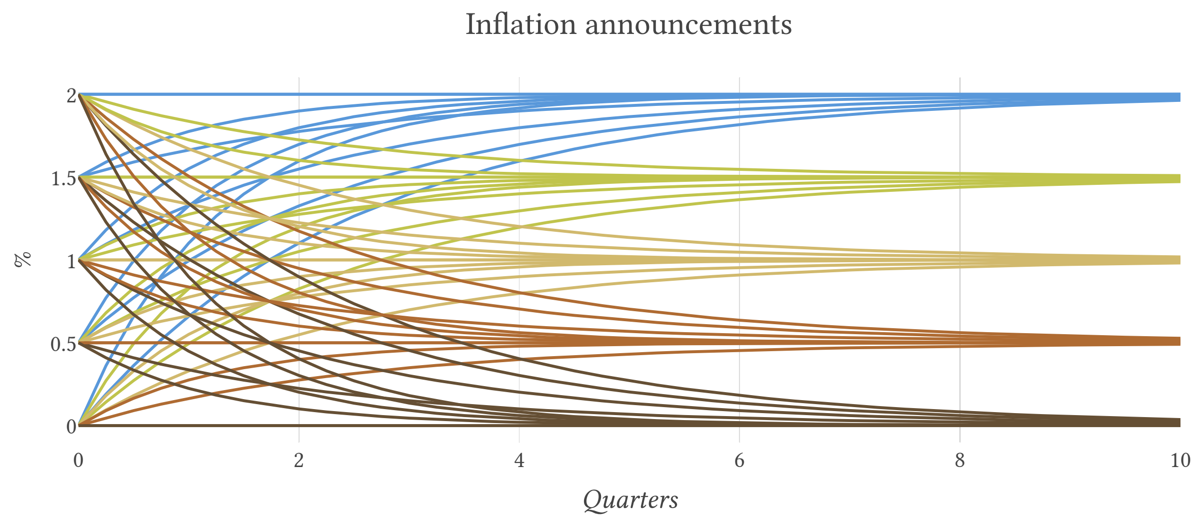


Figure 1: Possible behavioral types' announcements

2.2 *Timing of play*

At time 0 the government announces an inflation plan $a \in \mathcal{A}$. Then at each $t \geq 0$, the government chooses g_t and inflation π_t is given by (3). After inflation is realized, the private sector's beliefs are updated (see below) and output is determined by (2).

If the government is a behavioral type $c \in \mathcal{C}$, it announces $\left(a_t^c\right)_t$ and follows the announcement by setting $g_t = a_t^c$ at each time t. A rational government instead makes a strategic choice of announcement $r \in \mathcal{A}$ and is also free to choose any action g_t at time t based on time-t information, regardless of announced inflation a_t^r.

2.3 *Beliefs*

We refer to the probability the private sector attaches to the government being a behavioral type as its *reputation*. Reputation evolves as the private sector updates its beliefs via Bayes' rule following the initial announcement of targets at time 0 as well as realized inflation at each time t.

Consider the posterior beliefs following an announcement $a \in \mathcal{A}$. If $a \notin \mathcal{C}$ the private sector infers that the government is rational. If $a \in \mathcal{C}$, the government can be either rational or the behavioral type a. Suppose that in equilibrium the rational type announces a with density $\mu(a)$.

Then the posterior probability that the government is behavioral conditional on an announcement a with $\mu(a) > 0$ is given by

$$p_0(a; z, \mu) = \frac{z\nu(a)}{z\nu(a) + (1-z)\mu(a)}. \tag{4}$$

At time t, the private sector's posterior of the government being a behavioral type $c \in \mathcal{C}$ is formed by applying Bayes' rule to the private sector's information. Suppose that inflation π_t is realized at time t. If the government is behavioral of type c, then it must have chosen $g_t = a_t^c$ and the current shock must have been $\epsilon_t = \pi_t - a_t^c$, which has density $f_\epsilon(\pi_t - a_t^c)$. Let $g_t^\star$ denote the rational type's equilibrium strategy. Then if the government is rational, it must have chosen $g_t = g_t^\star$ so the shock must have been $\epsilon_t = \pi_t - g_t^\star$. Therefore, updating from a prior belief of p_t, we have that

$$p_{t+1} = \frac{p_t \cdot f_\epsilon(\pi_t - a_t^c)}{p_t \cdot f_\epsilon(\pi_t - a_t^c) + (1-p_t) \cdot f_\epsilon(\pi_t - g_t^\star)}.$$

It is useful to rewrite this condition as

$$p_{t+1} = p_t + p_t(1-p_t)\frac{f_\epsilon(\pi_t - a_t^c) - f_\epsilon(\pi_t - g_t^\star)}{p_t f_\epsilon(\pi_t - a_t^c) + (1-p_t) f_\epsilon(\pi_t - g_t^\star)}, \tag{5}$$

which shows that reputation moves slowly when it started close to 0 and 1, i.e. when the private sector is almost certain of the government's type. Large increases in reputation occur when realized inflation is much closer to the target than to the rational type's strategy, and large decreases in reputation require the converse. Consequently, large movements in reputation are more likely when the target differs significantly from the expected behavior of the rational type.

2.4 *Bellman equations after an announcement*

Given an announcement $c \in \mathcal{C}$, the problem of the rational type is to choose mean inflation g_t in period t to maximize (1) subject to (2), (3), and (5). The time-t government makes its choices taking as given its reputation p_t and the private sector's expectations, including the expected choice $g_t^\star$ of the rational type. We focus on Markovian strategies with $g_t^\star = g^\star(p_t, a_t^c)$. Thus, at time t the private sector expects the behavioral type to choose a_t^c and the rational type to choose $g^\star(p_t, a_t^c)$.

This allows us to write the rational government's problem recursively as

$$\mathcal{L}^c(p,a) = \min_g \mathbb{E}\left[(y^\star - y)^2 + \gamma\pi^2 + \beta\mathcal{L}^c(p', \phi_c(a))\right] \tag{6}$$

$$\text{subject to } \pi = g + \epsilon$$

$$\pi = \kappa y + \beta\big[p'\phi_c(a) + (1-p')g^\star(p', \phi_c(a))\big]$$

$$p' = p + p(1-p)\frac{f_\epsilon(\pi - a) - f_\epsilon(\pi - g^\star(p,a))}{pf_\epsilon(\pi - a) + (1-p)f_\epsilon(\pi - g^\star(p,a))}$$

Problem (6) illustrates the government's reputation-building incentives. By controlling current inflation, the rational type can, on average, affect its future reputation p'. This changes the continuation value $\mathcal{L}^c(p', \phi_c(a))$ as well as current output (through the Phillips curve). The effect on output is given by

$$\frac{\partial y}{\partial \pi} = \frac{1}{\kappa}\left[1 - \beta\frac{\partial p'}{\partial \pi}\left(\phi_c(a) - g^\star(p', \phi_c(a)) + (1-p')\frac{\partial g^\star(p', \phi_c(a))}{\partial p'}\right)\right]. \tag{7}$$

Inflation affects current output through three different terms. The first term, $\frac{1}{\kappa}\cdot 1$, describes the direct effect through the Phillips curve.

Inflation also affects output through changes in the posterior belief p'. In our numerical simulations the rational type chooses higher inflation than the announcement, so higher inflation decreases its reputation on average. The second term, $\beta\frac{1}{\kappa}\left(-\frac{\partial p'}{\partial \pi}\right)(\phi_c(a) - g^\star(p', \phi_c(a)))$, captures the fact that a lower posterior belief moves expectations of future inflation away from the target $\phi_c(a)$ and toward the expected choice of the rational type $g^\star(p', \phi_c(a))$. The third term, $\beta\frac{1}{\kappa}\left(-\frac{\partial p'}{\partial \pi}\right)(1-p')\frac{\partial g^\star(p', \phi_c(a))}{\partial p'}$, captures the effect of reputation on the rational type's future action.

At low levels of reputation p, inflation expectations place most of the weight on the rational type's action $g^\star(p', \phi_c(a))$. If future governments are expected to value their reputation and choose inflation close to the target, then the current government has an incentive to conserve its reputation and, therefore, also stay close to its announcement.

3. Equilibrium

3.1 Continuation equilibrium following an announcement

Consider the game immediately following an announcement c. A solution to (6) describes the government's choices $g(p, a)$ as a function of $g^\star$, the private sector's expectations of its actions. As is common with rational expectations, our notions of equilibrium emphasize the dual role played by $g^\star$: as the government's decision rule, and as a description of the private sector's expectations, which restrain the government (Sargent, 1999).

Definition. Given an announcement $c \in \mathcal{C}$, a *continuation equilibrium* consists of a loss function $\mathcal{L}^c : [0, 1] \times A \to \mathbb{R}$ and a policy function $g^\star_c : [0, 1] \times A \to \mathbb{R}$ such that

1. The loss function $\mathcal{L}^c$ satisfies the rational government's Bellman equation (6) taking as given expectations $g^\star_c$.
2. $g^\star_c$ is the policy function that attains the maximum in the solution to (6).

A useful property of continuation equilibria follows from close observation of problem (6): given the decay and asymptote parameters, it is equivalent to start the plan at a different initial announcement a or to just have arrived at a current announcement a as the continuation equilibrium unfolded.

Computation. To find a continuation equilibrium, we proceed as follows. Given a guess of $g^\star$, we solve problem (6) to find the government's best-response function g. We then iterate on this operator until a fixed point is reached.

Observation. Suppose $(\mathcal{L}, g^\star)$ is a continuation equilibrium for announcement $c = (a_0, \omega, \chi) \in \mathcal{C}$. Then for any b_0, the same pair $(\mathcal{L}, g^\star)$ is a continuation equilibrium for plan $c' = (b_0, \omega, \chi)$.

Another immediate property of equilibrium is that the government cannot design an inflation plan that will increase its reputation over time. If the rational government follows the announcement exactly, i.e. if $g^\star(p, a) = a$, its reputation will stay unchanged because inflation is not a signal of its type. And if the equilibrium strategy calls for deviations from the plan, reputation will decline on average. In other words, rational expectations do not allow the planner to accumulate reputation by consistently delivering on its promises, as such compliance would be anticipated by the private sector. What the planner can do is to design its plan in a way that provides incentives to deliver on it.

Observation. In any continuation equilibrium, the rational type's reputation is a supermartingale:

$$\mathbb{E}\left[p_{t+1} \mid \text{rational}, \mathcal{F}_t\right] \leq p_t,$$

where $\mathcal{F}_t$ denotes information up to time t. Thus, the planner cannot design an announcement that generates expected reputational gains in the ensuing continuation equilibrium.

3.2 *Equilibrium from the announcement stage*

An equilibrium of the entire game an initial announcement, as well as a set of continuation equilibria.

Definition. Given an initial reputation z, a *reputational equilibrium* is a distribution μ_z over $\mathcal{C}$ along with continuation equilibria $\{\mathcal{L}^c, \mathrm{g}^\star_c\}_{c \in \mathcal{C}}$ and a posterior reputation $p_0 : \mathcal{C} \to [0, 1]$ such that

1. Posterior reputation at each plan c satisfies Bayes' rule (4) given z and the distribution μ_z.

2. The distribution of mimicked types μ_z minimizes the posterior reputation-adjusted loss function

$$\mathcal{L}^\star(\mu_z, z) = \int_{\mathcal{C}} \mathcal{L}^c(p_0(c), a_0^c) d\mu_z(c)$$

   taking as given the posterior reputation function p_0.

The second part of the definition implies that the planner is indifferent among plans in the support of μ_z and prefers them to other plans:

$$\begin{aligned} &\mathcal{L}^c(p_0(c), a_0^c) = \mathcal{L}^{c'}(p_0(c'), a_0^{c'}) && \text{for } c, c' \in \text{supp}(\mu_z) \\ &\mathcal{L}^c(p_0(c), a_0^c) \leq \mathcal{L}^{c'}(1, a_0^{c'}) && \text{for } c \in \text{supp}(\mu_z),\ c' \notin \text{supp}(\mu_z), \end{aligned}$$

where we highlight the fact that announcements which are not made in equilibrium grant full reputation: $p_0(c) = 1$ for all $c \notin \text{supp}(\mu_z)$.

A lot of our analysis will focus on the limiting reputational equilibrium announcement as initial reputation vanishes:

$$\mu^\star = \lim_{z \to 0} \mu_z.$$

This can be interpreted as a refinement of the game where the government is known to be rational, demonstrating the optimal announcement when the public has little trust in the government's commitment.

An alternative definition of equilibrium follows Kambe (1999) and does away with the initial inference by the private sector. Instead, it corresponds to the case where the government first announces a plan c and subsequently becomes committed to following it with some exogenous probability p_0, independent of c.

Definition. For given $p_0 \in [0, 1]$, a K-*equilibrium* is an announcement $c_K(p_0)$ with an associated continuation equilibrium $\{\mathcal{L}^K, g_K\}$ such that

$$c_K(p_0) = \arg\min_{c \in \mathcal{C}} \mathcal{L}^c(p_0, a_0^c)$$

Similarly to reputational equilibria, we are especially interested in the limiting announcement $c_K(p_0)$ as $p_0 \to 0$.

3.3 *Credibility*

While the government's reputation describes the likelihood it is committed to the plan, it does not reflect how closely the plan is followed on average across both types. To obtain a measure of this, we define the credibility of a plan as the ratio of announced and (expected) realized inflation, normalized by their distance from Nash inflation.

Definition. Given a plan c, its *remaining credibility* in state (p, a) is defined recursively as follows:

$$\begin{aligned} C(p, a; c) &= \mathbb{E}\left[(1-\beta)\frac{\pi^N - \pi}{\pi^N - a} + \beta C(p_c'(p, a), \phi_c(a); c)\right] \\ &= (1-\beta)\frac{\pi^N - [pa + (1-p)g_c^\star(p, a)]}{\pi^N - a} + \beta\mathbb{E}\left[C(p_c'(p, a), \phi_c(a); c)\right] \qquad (8) \end{aligned}$$

where π^N is Nash inflation. The *credibility* of a plan in a K-equilibrium is given by

$$C^K(c) = \lim_{p \to 0} C(p, a_0(c); c),$$

while in a reputational equilibrium it is

$$C^\star(c) = \lim_{z \to 0} \int C(p_0(c), a_0(c); c) d\mu_z(c).$$

In our simulations $g_c^\star(p, a) \in [a, \pi^N]$ for all (p, a) and all plans c, so credibility lies in $[0, 1]$.

4. Analysis and Numerical Results

We solve the model numerically for different announcements $c \in \mathcal{C}$.

4.1 *Parametrization*

We parametrize our model following Lu, King, and Pastén (2016). Our preference and technology parameters $\gamma, \kappa, y^\star$ are consistent with the planner's objective function and Phillips curve in a standard New Keynesian economy calibrated to US data (Galí, 2015; Galí and Gertler, 1999). Table 1 summarizes our parameter choices. These parameters imply a level for Nash inflation of about

Table 1: Benchmark calibration

Parameter	Value	Definition	Source / Target
β	0.995	Discount factor	2% real interest rate
γ	60	Inflation weight	Lu, King, and Pastén (2016)
σ	1%	Std of control shock	Lu, King, and Pastén (2016)
κ	0.17	Slope of Phillips curve	Lu, King, and Pastén (2016)
$y^\star$	5%	Output target	Lu, King, and Pastén (2016)

2% annualized.

4.2 *Continuation equilibrium after announcement* c

Figure 2 shows a typical value function $\mathcal{L}^c(p, a)$ for an arbitrary plan c. All plots have current reputation p on the x-axis. Darker lines correspond to a lower current target a, which is measured relative to Nash inflation π^N.

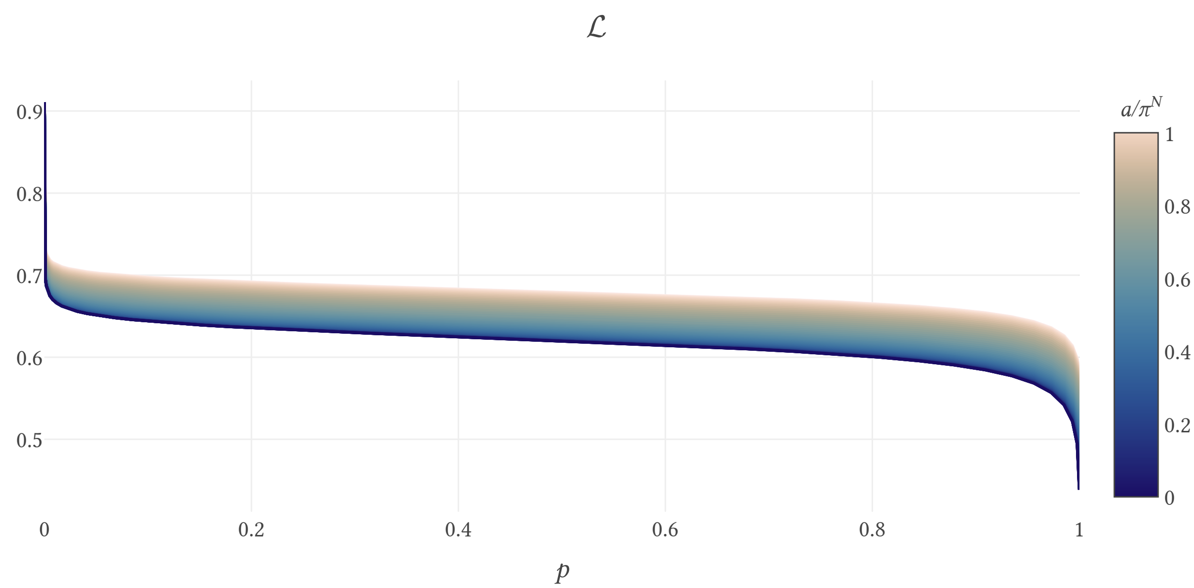


Figure 2: Loss function after announcement c

A few observations are in order. Firstly, $\mathcal{L}^c$ is decreasing in p. An increase in reputation generally decreases expected inflation leading to higher current output.

Secondly, the loss function has a convex-concave shape reflecting the dynamics of reputation. When reputation is close to 0 or 1 the public is confident in its assessment of the government's type, and significant evidence is required to move beliefs. Thus, marginal changes in reputation are more valuable leading to the steepness of the loss function.

Thirdly, at high levels of reputation, a lower target a is preferred. This is because the public's expectations are driven mostly by the behavioral type, who sticks to the announcement.

Finally, the range of values across different targets is generally smaller at lower levels of reputation. One reason is that with lower reputation the current target becomes less relevant, as its weight in expected inflation decreases. Another, more nuanced reason is that the tradeoffs between stimulating output and sustaining reputation are more pronounced when the government is unlikely to be committed to the target. While lower targets are directly beneficial to inflation expectations, the government finds large deviations from these targets optimal, resulting in larger reputational losses (see discussion below). Thus, the benefits of ambitious announcements with low targets can be offset more heavily at low levels of reputation.

Figure 3 shows how the current target a and reputation p affect $g^\star(p, a) - a$, the (rational) government's deviation from the target. The deviation is generally decreasing in reputation, except when both reputation and the announcement are near zero. There is a discontinuity at zero reputation, where the unique Markov equilbirium exhibits inflation π^N regardless of the announcement, since there are no reputational concerns. In contrast, even at small levels of reputation, the government can benefit from staying close to the announcement.

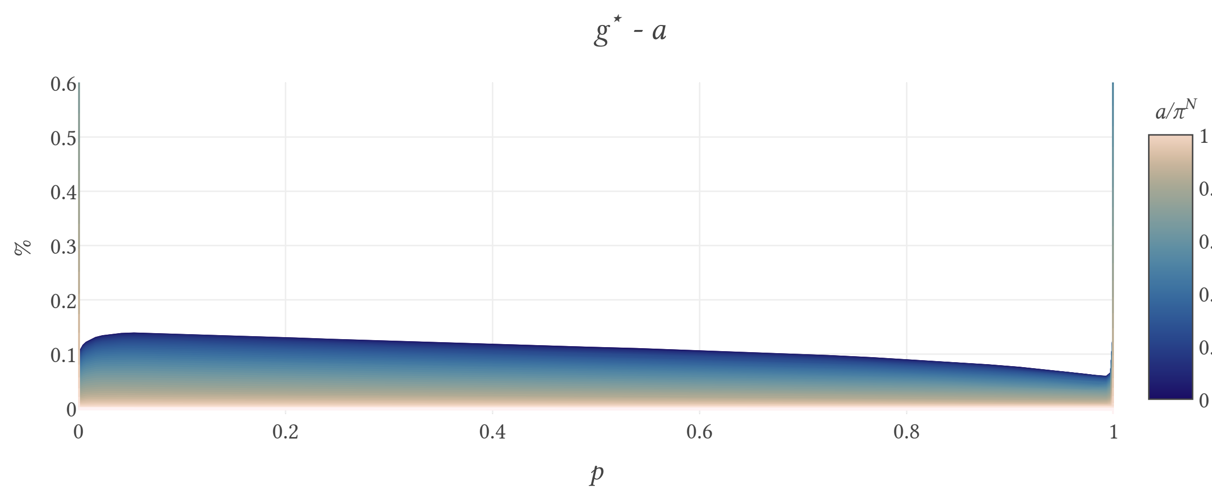


Figure 3: Inflation deviations

The effect of announcements on deviations is unambiguous: a lower target a causes the rational type of the government to deviate further from it. In comparison to an equilibrium with a higher target, the government needs to inflate less to maintain the same evolution of reputation, holding the public's expectations of its policy fixed. Thus, there are lower costs to creating surprise inflation when the target is low.

Current reputation p has a number of effects on deviations from the target. On the one hand, a larger stock of reputation makes the planner more inclined to spend it by creating substantial surprise inflation. On the other hand, higher reputation anchors expectations more tightly and makes it less costly for the government to preserve its reputation by staying close to the target, especially when the target is high.

Figure 4 shows average reputation p' as function of current reputation p and target a. As

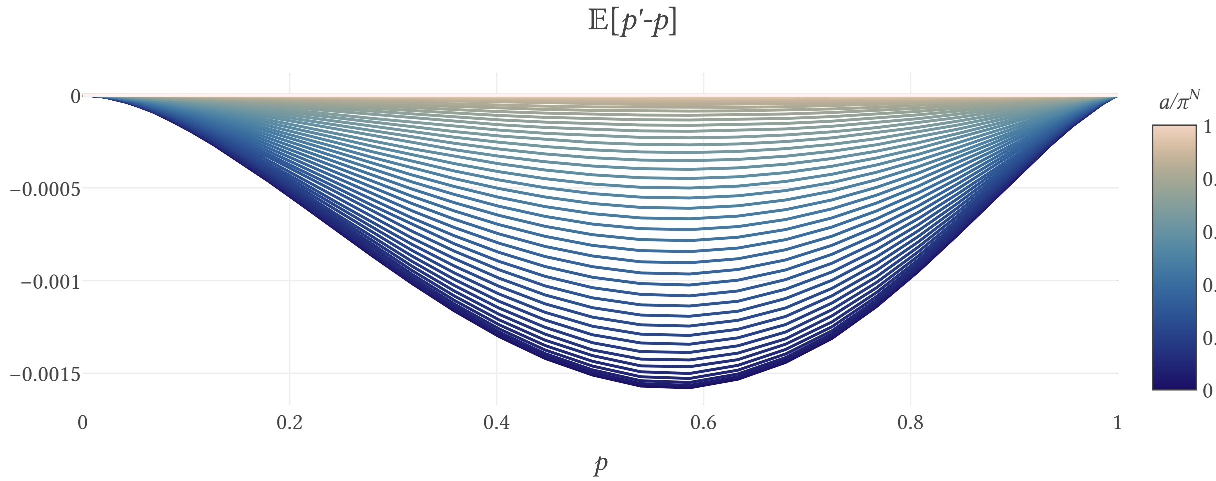


Figure 4: Expected reputation losses

previously noted, the government's reputation declines on average: $\mathbb{E}[p']$ is never above p. In particular, no reputation is lost only when the current target a corresponds to Nash inflation π^N, where incentives to create surprise inflation are eliminated. Also, as a consequence of Bayes' rule, changes in reputation are smaller when initial reputation is closer to 0 or 1. Finally, lower announcements are associated with a larger expected reputation loss. Lower, more ambitious targets generate weaker incentives: as the temptation to inflate grows larger, the government prefers to spend more of its reputation to achieve higher output.

4.3 K-*equilibrium announcements*

Figure 5 shows K-equilibrium announcements. Each announcement minimizes the government's loss function conditional on starting with reputation p_0. The top panel shows the decay rate $1 - e^{-\omega}$ (in percent terms), while the bottom panel shows the choice of initial inflation a_0 and asymptote χ.

At $p_0 = 1$, any announcement is believed by the private sector, regardless of expectations about the behavior of the rational type. The government sets expectations at their most advantageous level by promising zero inflation throughout. Since the private sector is convinced

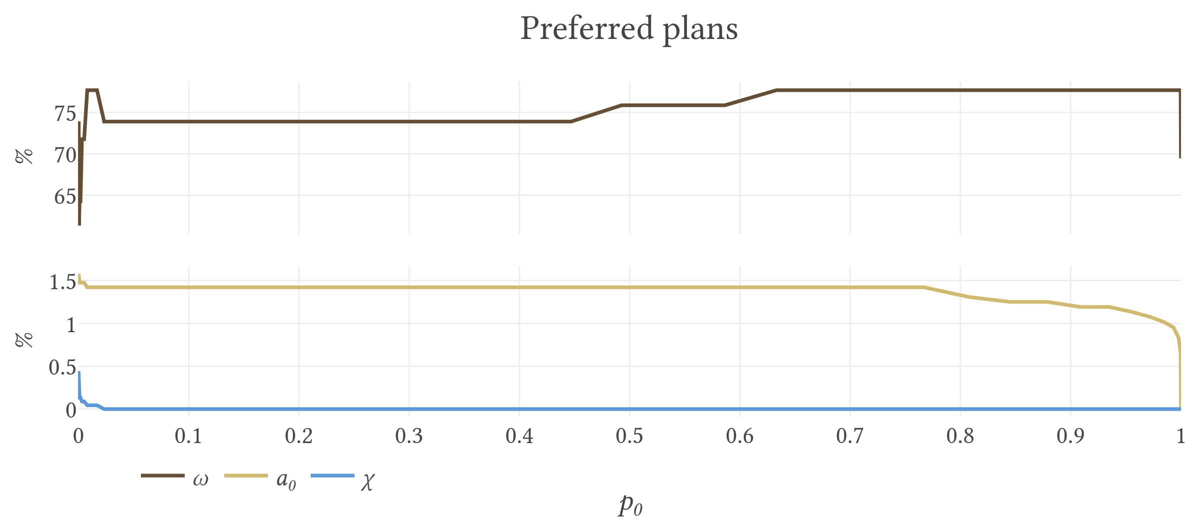


FIGURE 5: K-EQUILIBRIUM ANNOUNCEMENTS

the government is committed to the announcement, the rational type has a clear incentive to create surprise inflation. When $p_0 < 1$, such a deviation incurs reputational losses. Thus, the government announces plans with inflation above zero, evidenced by the initial target $a_0 > 0$. Positive inflation targets help the incentives of the government's future selves to stay closer to the announcement, conserving reputation. Since $a_0 > \chi$, the optimal inflation targets decrease over time as the government's reputationa declines stochastically. This structure of the optimal announcements is preserved even as initial reputation p_0 approaches zero.

Figure 6 shows the determination of the K-equilibrium when p_0 is small. For each decay rate ω and asymptote χ we plot the minimized loss function $\min_{a_0} \mathcal{L}(p_0, (a_0, \omega, \chi))$. The minimum is achieved at a point with both $\omega > 0$ and $\chi > 0$: the K-equilibrium has the initial planner promise a gradual disinflation that does not converge to the first-best level of zero inflation.

When χ is small, plans eventually call for ambitiously low levels of inflation which are difficult to sustain. Consequently, reputation is quickly lost giving rise to unfavorable continuation values. This makes slower decay rates preferable pushing promises of low inflation to the far future. On the other hand, high values of χ make the late stages of the plan easy to keep but provide little benefit as they promote high expectations.

The optimal plan involves an intermediate announcement of long-run inflation, balancing the desire for low expectations with incentives to preserve reputation. It features a high decay rate

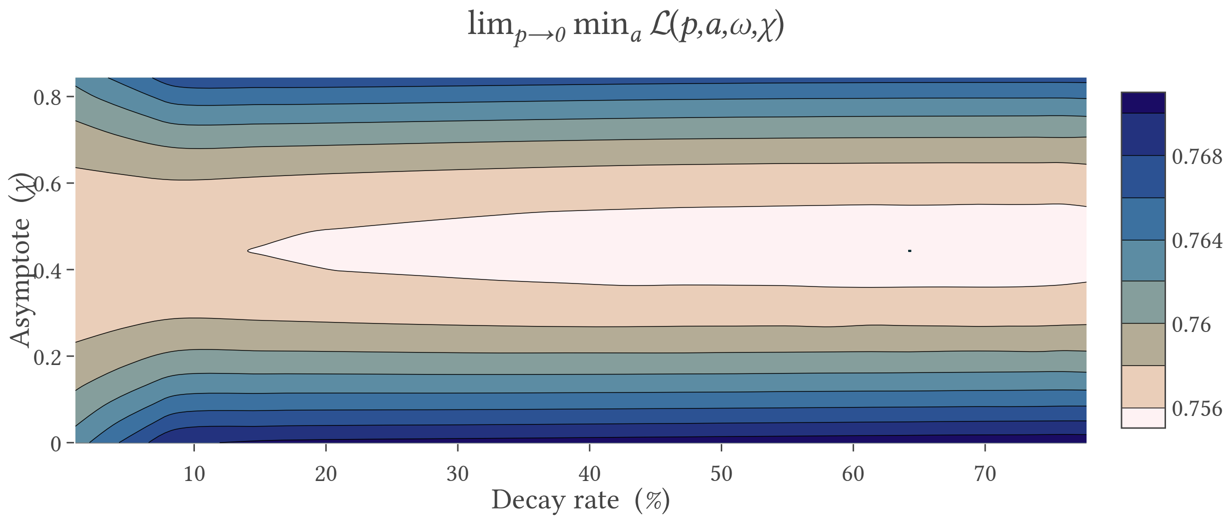


FIGURE 6: LOSS FUNCTION ACROSS ANNOUNCEMENTS

compared to optimal plans for extreme values of χ. The steep descent of inflation lowers inflation expectations relative to the current target due to the forward-looking nature of the Phillips curve, strengthening short-run incentives to stick to the announcement.

4.4 *Credibility*

Our setup distinguishes reputation p, the posterior that the government is the behavioral type that was announced, from credibility $C(p, a; c)$, the expected discounted deviations from plan c at reputation p and current target a, as defined in (8). Figure 7 plots the credibility of different plans at vanishingly small reputation, as a function of the decay rate ω and the asymptote χ, for the loss-minimizing initial inflation a_0 at those parameters.

Plans with a lower asymptote are less credible, as are plans with a steeper promised descent of inflation. One should be careful about this result, as it mostly reflects the fact that plans with fast decay reach a phase in which the targets are almost constant more quickly.

4.5 *Distribution of announcements in the reputational equilibrium*

We now turn to a description of the reputational equilibrium and the associated distributions μ_z of announcements by the rational type. Figure 8 plots the average plan as a function of initial

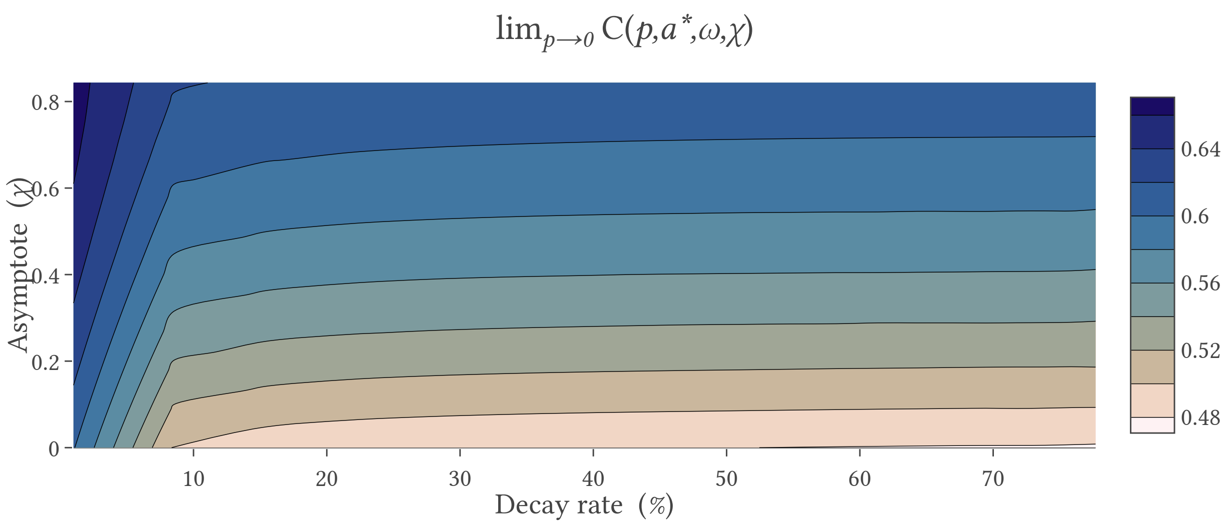


FIGURE 7: CREDIBILITY

reputation z prior to the announcement.

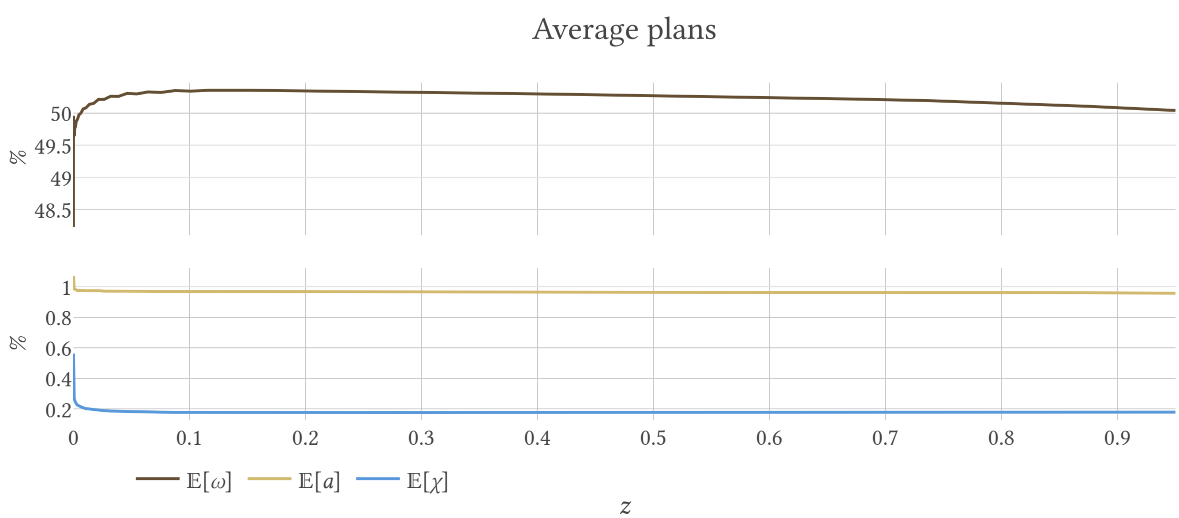
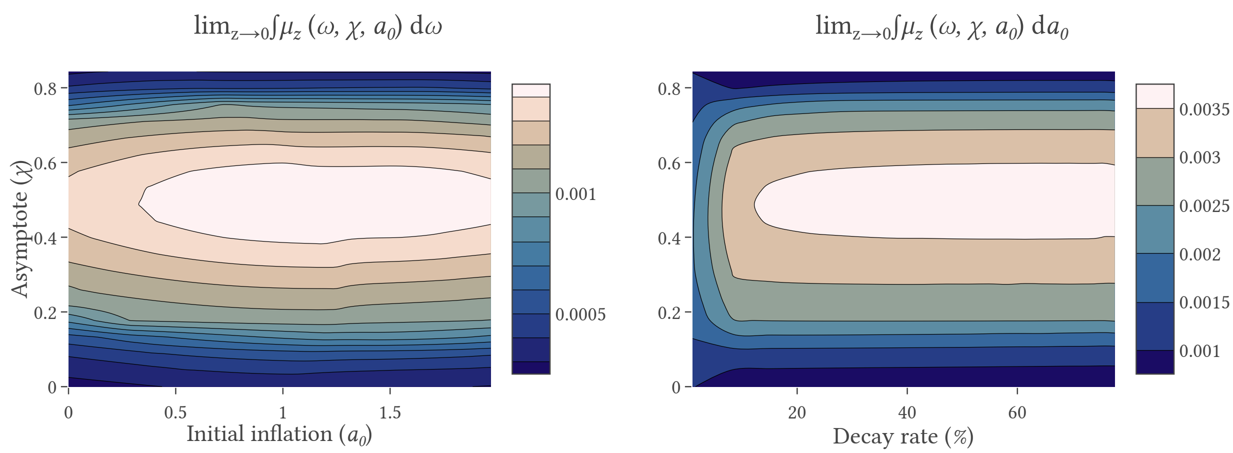


FIGURE 8: REPUTATIONAL EQUILIBRIUM ANNOUNCEMENTS

For intermediate values of initial reputation, the planner chooses (on average) a disinflation path that starts from about half of Nash inflation π^N and converges towards a tenth of it by about half the distance each period. As initial reputation becomes small, the planner starts to put more weight on plans that converge toward a higher asymptote χ, similarly to the optimal K-equilibrium announcement in Figure 5.

Figure 9 shows the limiting distribution $\mu^\star = \lim_{z\to 0} \mu_z$ of announcements as initial reputation vanishes. The left panel shows the distribution of types as a function of the asymptote χ and initial inflation a_0, integrating over the decay rate ω, while the right panel integrates over initial inflation. Figure 9a shows that the government tends to choose gradual plans with higher initial



(A)

(B)

FIGURE 9: DISTRIBUTION OF TYPES

inflation a_0 than asymptote inflation χ. This probability is $\mathbb{P}(a_0 > \chi) = 71.1\%$. Plans with initial inflation of even five times the asymptote are still announced quite often: $\mathbb{P}(a_0 > 5\chi) = 17.9\%$. While the level of initial inflation varies from plan to plan, the asymptote seems more precisely set: the density of a_0 and χ in the reputational equilibrium announcement falls sharply for χ away from the optimum, while it stays flat over many more values of initial inflation a_0.

On the right, Figure 9b bears a close resemblance to Figure 6 which plotted the K-equilibrium loss function at low p_0. Announcements with a lower loss in the K-equilibrium are good for the planner, so they are chosen more often in the reputational equilibrium. Hence, the government starts with lower reputation in those plans, which lowers their value in the equilibrium. This initial update of reputation (from z to p_0) makes the planner indifferent across all equilibrium announcements, which ultimately justifies the mixed strategy. Similarly to the K-equilibrium, there is not much variation in the value of announcements along the decay rate dimension, evidenced by the nearly flat density $\mu^\star$. One exception is that very small decay rates are significantly less desirable, in contrast to the K-equilibrium.

5. Comparative Statics

Figure 10 shows the average plan announced in the reputational equilibrium as a function of the variance of the control shock σ around its baseline value of 1%. More noise in the control makes deviations from targets harder to detect. Therefore, the level of adherence to plans is decreasing in σ. This makes the planner choose less ambitious plans when the control over inflation is less tight. These plans have higher inflation throughout, as they feature a higher asymptote χ, a marginally higher initial inflation a_0, and slower decay ω.

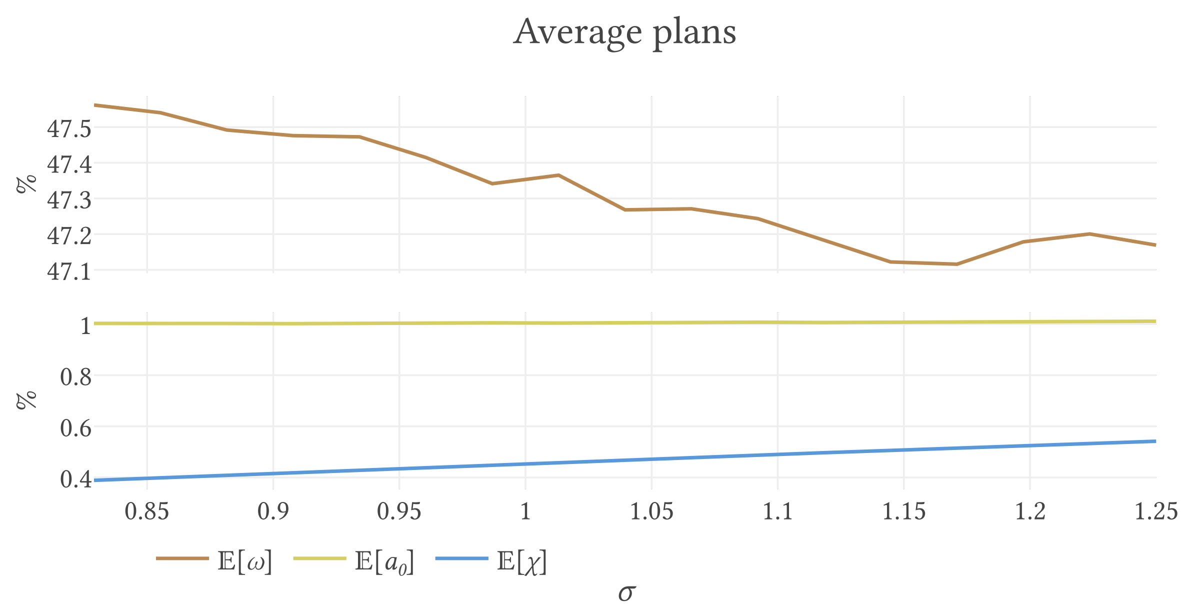
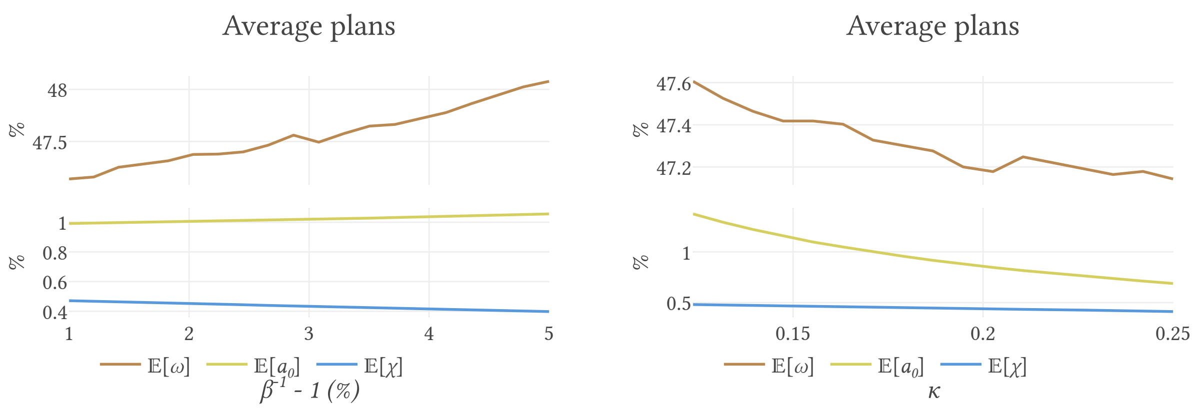


Figure 10: Average plans and the control shock variance

Figure 11 repeats the exercise varying the discount factor β and the slope of the Phillips curve κ. It reveals some subtleties in the manipulation of the three parameters that describe our plans. Figure 11a shows the average plan as a function of the discount rate $1/\beta - 1$ (whose benchmark value is 2% in annual terms). As the planner becomes more impatient, average plans start higher but converge to lower inflation, with a faster decay rate. With more impatience, the public expects a larger inflation bias. For this reason, the planner tends to choose plans that are more resilient. Increasing initial inflation makes the plan easier to keep, while decreasing asymptotic inflation makes it more costly to deviate early on. Having a steeper descent of inflation contributes to both objectives.

Figure 11b shows that when the slope of the Phillips curve is increased from its baseline value of 0.17, the planner announces lower inflation throughout. When the Phillips curve is steeper, there are weaker incentives to create surprise inflation, as it results in a smaller output boom. Thus, the planner lowers expectations through lower targets without increasing reputational losses.



(A) Varying the discount factor

(B) Varying the slope of the Phillips curve

Figure 11: Average plans

6. Other Models

Our benchmark model of reputation with imperfect control yields gradual disinflation plans. We dissect this result by comparing our model to salient models in the literature on sustainable plans. We start from the Ramsey plan and slowly enrich the structure of the model to see which mathematical features create the incentive for gradualism.

6.1 *The Ramsey plan*

We refer to choosing an entire path $\{g_t\}_t$ with commitment to minimize (1) subject to (2) and (3) as the *Ramsey* plan. In this case there are no announcements: with commitment, the planner chooses the public's expectations directly, which coincide with its own future actions. The linear-quadratic structure of this problem alows us to disregard the issue of imperfect control:

the presence of the shocks ϵ_t only affects the planner through a variance term independent of policy.

Standard techniques allow us to use Lagrange multipliers of each time-t constraint to write the problem recursively for $t > 0$ as

$$v^{FB}(\theta) = \max_{\theta'} \min_{y,\pi} (y - y^\star)^2 + \gamma\pi^2 + \theta'(\pi - \kappa y) - \theta\pi + \beta v^{FB}(\theta') \tag{9}$$

Problem (9) produces policy functions $g_\pi^{FB}(\theta), g_y^{FB}(\theta), g_\theta^{FB}(\theta)$ that describe the planner's actions for each period. At time 0, the planner does not carry any multipliers from the past and attains a value

$$\mathcal{J}^{FB} = v^{FB}(0) \tag{10}$$

To reconstruct the plan, we recursively apply the policy functions g^{FB}, starting from the solution θ^{FB} to (10)

$$\theta_t = \begin{cases} g_\theta^{FB}(\theta_{t-1}) & \text{if } t > 0 \\ 0 & \text{if } t = 0 \end{cases} \qquad \text{and} \qquad \pi_t = g_\pi^{FB}(\theta_t) \tag{11}$$

As is well known, time inconsistency manifests itself in the fact that $g_\theta^{FB}(0) \neq 0$: a free planner wishes its future self to face multipliers that curtail the choice of inflation.

Figure 12 plots the Ramsey plan against the average announcement in the reputational equilibrium, as well as the announcement in the K-equilibrium. The power of commitment enables a complete disinflation: Ramsey inflation is essentially zero a year and a half after the announcement of the plan. The Ramsey plan also starts with a high initial target, about three quarters of the way to Nash inflation. This is because committing to inflation for period t constrains the planner by determining expected inflation at $t - 1$, but this constraint is absent in the initial period. Inflation then comes down as the planner smooths the benefits of initial inflation on output over a few of the initial periods.

In contrast, the equilibrium announcements do not converge to zero. Even in a reputational equilibrium where the planner randomizes between announcements, the probability of plans with low long-run inflation, i.e. $\chi \approx 0$, is low (Figure 9a). Such plans are avoided due to their low credibility. The K-equilibrium announcement that minimizes the loss conditional on starting from

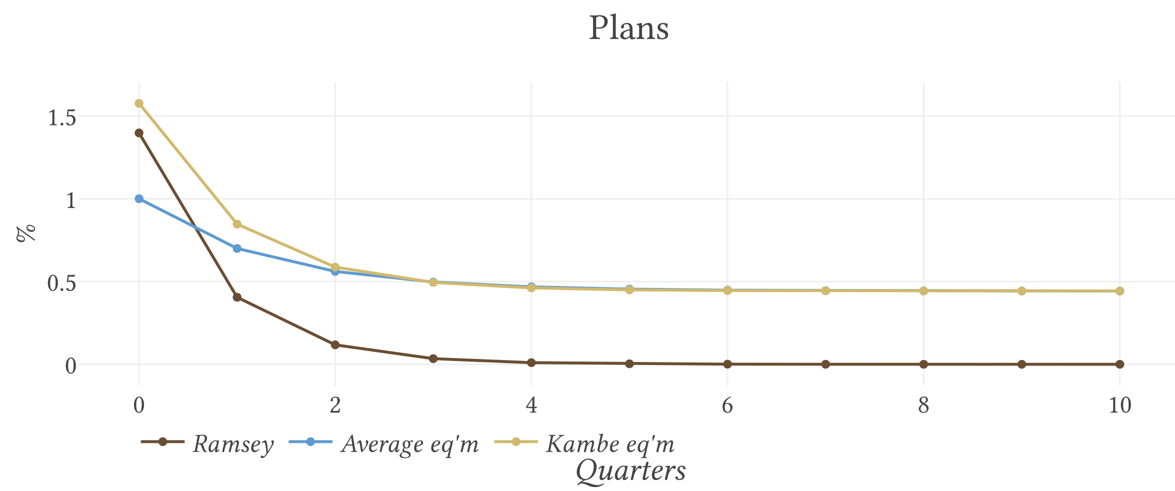


FIGURE 12: THE RAMSEY PLAN AND EQUILIBRIUM ANNOUNCEMENTS

a very low level of reputation, starts above the Ramsey plan. It needs high initial inflation to create the descent that fuels incentives. In a reputational equilibrium, some plans allow the planner to start with a higher reputation as they are less likely to be chosen in equilibrium. These plans tend to exhibit higher inflation than the K-equilibrium announcements in order to maintain the indifference of the planner. This ultimately causes the average reputational equilibrium announcement to start above the K-equilibrium announcement.

6.2 *Sustainable plans with expectations as threats*

The Ramsey plan can be decentralized by choosing appropriately high inflation expectations ξ in case of deviation. Consider a setting where control over inflation is perfect, so any deviation is detected by the private sector with certainty. A natural counterpart to reputation is an indicator $p \in \{0, 1\}$ of whether the planner has maintained the promised path of inflation so far. Then the optimal sustainable plan can be written recursively as follows:

$$v^{\xi}(p, a) = \min_{y,\pi,a'} \; (y - y^{\star})^2 + \gamma\pi^2 + \beta v^{\xi}(p', a') \tag{12}$$

$$\text{subject to} \quad \pi = \kappa y + \beta \left(p' g_{\pi}^{\xi}(a') + (1 - p')\xi \right)$$

$$p' = \begin{cases} 1 & \text{if } \pi = a \text{ and } p = 1 \\ 0 & \text{otherwise} \end{cases}$$

where $g_\pi^\xi(a)$ is the equilibrium policy at state a under punishment ξ, which the planner takes as given but coincides with its v^ξ-minimizing choice. In the first period, the planner has full reputation ($p = 1$) and is allowed to choose the initial inflation, attaining value

$$\mathcal{J}^\xi = \min_a v^\xi(1, a) \tag{13}$$

Figure 13 plots sustainable plans for different values of ξ, along with the Ramsey plan. We re-

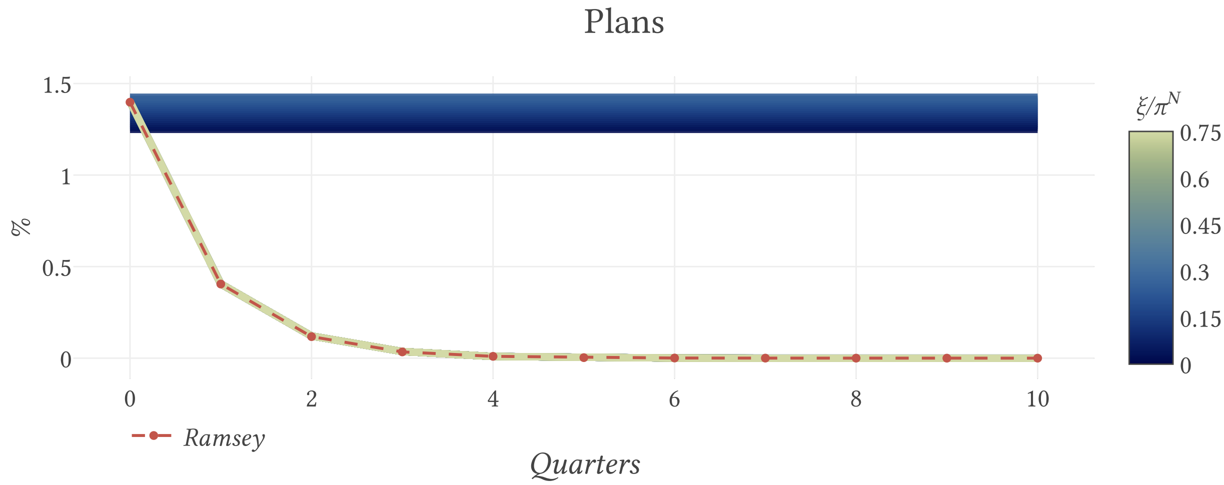


Figure 13: Sustainable plans at different punishments

cover a well-known result from the literature: when the punishment is harsh enough, the Ramsey plan is sustainable. When the Ramsey plan is not sustainable, the planner deviates immediately and ξ only affects the level of inflation expectations the planner is best-responding to.

6.3 *Sustainable plans with reverting triggers*

The Ramsey plan is built on the fact that all deviations are detected. In this case, the private sector can threaten to, in our language, send the reputation to zero. With imperfect control, this is no longer the case. The private sector understands that any threat might be triggered on-path without actual deviations having taken place. Assessing punishment strategies needs to weigh deterrent against the costs from false positives. This is why we consider a version in which, like in Green and Porter (1984), the private sector shifts to a punishment regime whenever realized inflation deviates from the target by more than some threshold, and reverts stochastically from it.

An equilibrium with reverting triggers is parametrized by a distance D between realized and announced inflation that triggers the punishment, a probability of return θ to the normal regime, and 'punishing' expectations ξ. We have

$$v^G(a) = \min_{g,a'} \mathbb{E}\left[(y - y^\star)^2 + \gamma\pi^2 + \beta\left(p' v^G(a') + (1-p')v^P\right)\right] \tag{14}$$

$$\begin{aligned} \text{subject to} \quad & \pi = g + \epsilon \\ & \pi = \kappa y + \beta\left(p' g^G(a') + (1-p')\xi\right) \\ & p' = \begin{cases} 1 & \text{if } \frac{|\pi - a|}{a} < D \\ 0 & \text{otherwise} \end{cases} \end{aligned}$$

where

$$v^P = \min_{\pi,a'} (y - y^\star)^2 + \gamma\pi^2 + \beta\left(\theta v^G(a') + (1-\theta)v^P\right) + \sigma_\epsilon^2\left(\gamma + \frac{1}{\kappa^2}\right) \tag{15}$$

$$\text{subject to} \quad \pi = \kappa y + \beta(\theta a' + (1-\theta)\xi)$$

since the control shock enters only through a variance term, as in the Ramsey plan. In the first period the planner can choose the initial announcement in the G regime ($p = 1$) attaining a value

$$\mathcal{J}^G = \min_a v^G(a) \tag{16}$$

Figure 14 shows the path of mean inflation $g^G(a_t)$ for different levels of ξ assuming that the punishment regime is never triggered.

As before, when ξ is small the planner understands that promises will be broken and simply best responds to these expectations. When ξ is larger, the planner is able to target lower levels of inflation. However, due to imperfect control the optimal announcement is neither zero inflation, nor the Ramsey plan; instead, the planner promises a constant intermediate level of inflation.

We also note that this setup can be made mathematically more similar to our other plans by

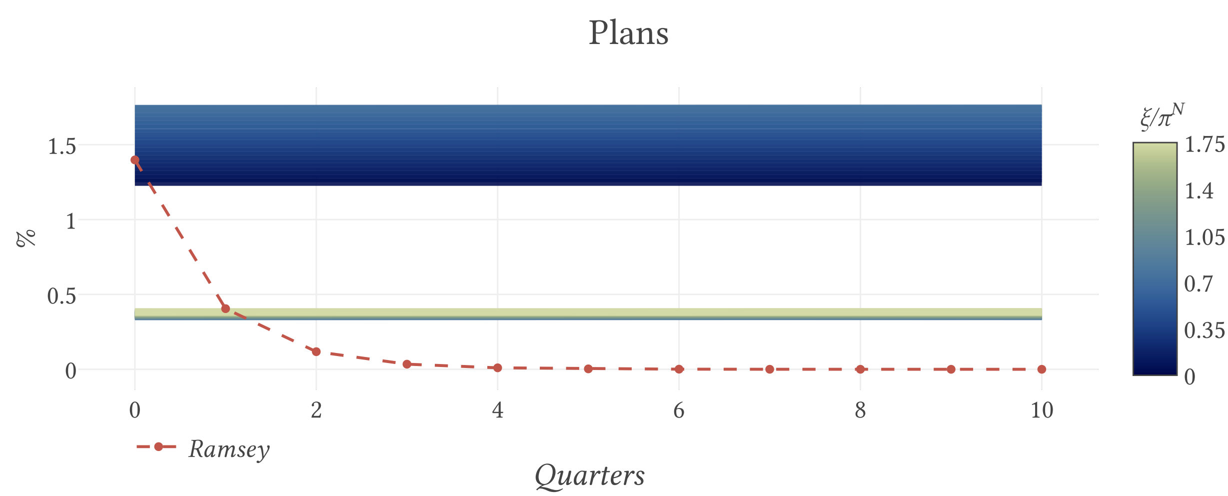


FIGURE 14: SUSTAINABLE PLANS WITH REVERTING TRIGGERS AT DIFFERENT PUNISHMENTS

appropriately modifying the constraints

$$v^{GP}(p,a) = \min_{g,a'} \mathbb{E}\left[(y-y^\star)^2 + \gamma\pi^2 + \beta\left(v^{GP}(p',a')\right)\right]$$

$$\begin{aligned}
\text{subject to}\quad & \pi = g + \epsilon \\
& \pi = \kappa y + \beta\left(p' g^{GP}(p',a') + (1-p')\xi\right) \\
& p' = \begin{cases} \begin{cases} 1 & \text{if } \frac{|\pi - a|}{a} < D \\ 0 & \text{otherwise} \end{cases} & \text{if } p = 1 \\ \begin{cases} 1 & \text{with prob } \theta \\ 0 & \text{with prob } 1-\theta \end{cases} & \text{if } p = 0 \end{cases}
\end{aligned}$$

6.4 *Recursive plans with reputation*

Notice that in all these previous versions, the roles of $g(a)$ and ξ are reversed in comparison to our baseline notions of reputational and K-equilibrium. In the Ramsey model and our version of the Green-Porter model, the private sector expects the government's strategy when reputation is high and the exogenous parameter ξ (the private sector's threat) when it is low. In both reputational and K-equilibria, the private sector expects the behavioral government to follow the announcement and the rational government to follow its strategy. While this may appear as an innocuous relabelling of types, we will now show that this shift makes the optimal plan look very different.

We now consider a version in which, like in Dovis and Kirpalani (2019), the government is made up of two agents: a morning planner and an afternoon policy maker. In the morning of period t, the planner announces a policy recommendation for the afternoon of $t+1$ (given an announcement for the afternoon of t made earlier). The action in each period happens in the afternoon, when the policy maker chooses inflation. The policy maker can be either a commitment type who stubbornly follows the recommendation, or a rational type who chooses whether to follow it. The planner and the private sector do not know which type of policy maker they face but make statistical inference based on past realizations of inflation. In contrast to Dovis and Kirpalani (2019), we retain the features of imperfect control and the New Keynesian Phillips curve from the baseline model. We refer to the equilibrium announcements in this game as *recursive plans (with reputation)*.

Our timing assumption is done for simplicity and to improve comparability to our benchmark. The target at $t+1$ can depend on reputation at t but not on realized inflation at t, or reputation at $t+1$. Thus, when faced with a target a and its current reputation is p, the policy maker attains a value of

$$v^R(p,a) = \min_{g,a'} \mathbb{E}\left[(y-y^\star)^2 + \gamma\pi^2 + \beta v^R(p',a')\right] \tag{17}$$

$$\begin{aligned} \text{subject to} \quad & \pi = g + \epsilon \\ & \pi = \kappa y + \beta\left(p'a' + (1-p')g^R(p',a')\right) \\ & p' = p + p(1-p)\frac{f_\epsilon(\pi-a) - f_\epsilon(\pi - g^R(p,a))}{pf_\epsilon(\pi-a) + (1-p)f_\epsilon(\pi-g^R(p,a))} \end{aligned}$$

The planner with initial reputation p then attains an initial value of

$$\mathcal{J}^R = \min_a v^R(p,a)$$

by making an optimal announcement. Recursive plans with reputation essentially coincide with our reputational equilibria, except that the timing assumptions allow the planner to announce the plan sequentially and not upfront like in our benchmark case. We discuss these differences below.

Figure 15 plots the recursive plan with reputation for different levels of initial reputation. The planner with full reputation enjoys complete credibility for all plans and hence announces

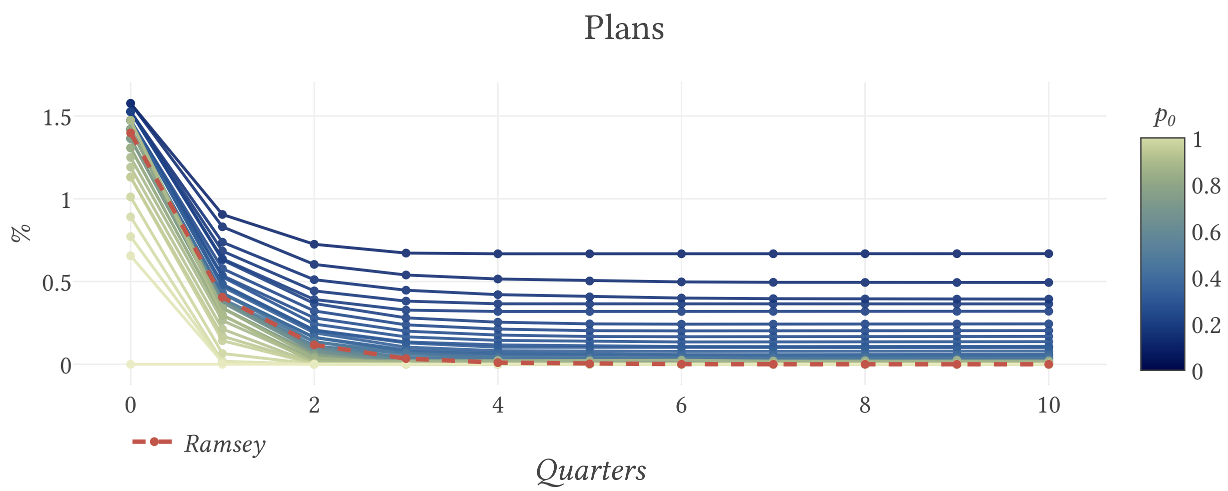


Figure 15: Recursive plans with reputation

a plan of zero inflation throughout. The fully-credible planner intends to break this plan as this will not alter expectations in any way. With moderate initial reputation the planner announces a positive level of initial inflation and a convergence to zero inflation which decreases in speed as initial reputation falls. When initial reputation is low, plans start to converge to higher long-run inflation, in a similar pattern to the one delivered by the K-equilibrium.

While recursive plans with reputation are inspired on Dovis and Kirpalani (2019), the models are very different. Theirs is predicated on considering perfect control of inflation as well as a traditional Phillips curve where output responds to the difference between realized and expected inflation for the current period. Both differences turn out to be important: adding noise while retaining a traditional Phillips curve, or considering a forward-looking Phillips curve without noise yields a flat optimal plan.

6.5 *Relation to other models*

In Section 6 we explored the individual role of the two main assumptions of our model: reputation and imperfect control (noise). Figure 16 summarizes related models from the literature. Our starting point is the Ramsey plan and its decentralization as a sustainable plan in the absence of reputation and noise. Barro (1986) and Backus and Driffill (1985) introduced reputation into sustainable plans, maintaining perfect control of inflation. They consider a behavioral type

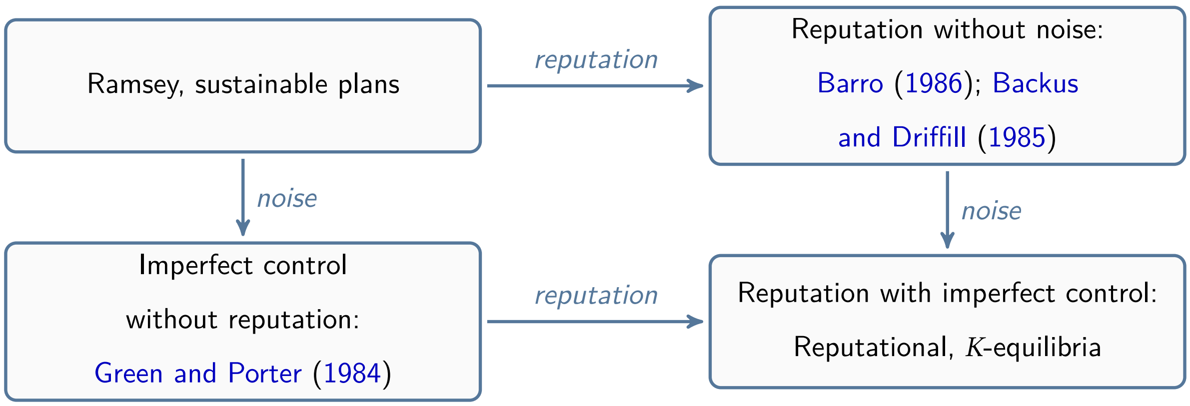


Figure 16: Models

who sticks to the Ramsey plan, which entails zero inflation throughout due to a traditional (not forward-looking) Phillips curve. The rational government follows the plan in all but the very last periods of an arbitrarily long horizon. Thus, even if other announcements were permitted, the Ramsey plan would remain optimal.

Our reformulation of the Green and Porter (1984) model in Section 6.3 introduces noise but leaves out reputation. In this case, punishing expectations that are high enough can induce the government to choose a plan with below-Nash inflation. However, all plans exhibit a constant level of inflation.

Our model of reputational equilibria incorporates both reputation and noise. Consequently, reputation moves smoothly and cannot be lost completely. The planner is then able to take advantage of the dynamics of targets, as they impact the dynamics of its reputation. Our reformulation in terms of recursive plans with reputation in Section 6.4 shows that the result does not depend on the planner being able to announce all targets at once.

6.6 *Gains from preannouncement and flexibility*

Figure 17 plots the recursive plan with reputation against some of the other plans we have considered so far.

Inspecting problems (6) and (17) reveals that, while very similar, the recursive plan differs from optimal plans in reputational and K-equilibria in two important ways. In the latter, inflation

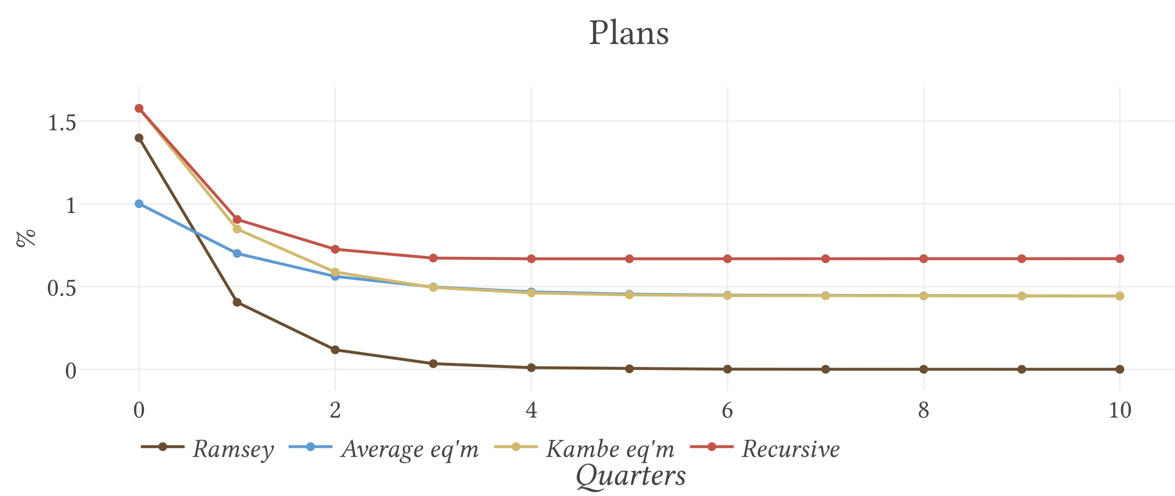


Figure 17: Plans with reputation

targets for all periods are chosen at the beginning of time, while in the recursive plan they are chosen sequentially. Consequently, the recursive plan allows for feedback between the evolution of reputation and future targets.

Because of these differences, comparing the recursive plans to our baseline notions of equilibrium will mix two distinct effects. On the one hand, because the recursive plan can respond to the evolution of reputation, it benefits from the *flexibility* of tailoring future announcements to the current assessment of the credibility of different plans. On the other hand, reputational and K-equilibrium plans benefit from *pre-announcing* the targets. This allows the planner to use the partial credibility of plans in the future to induce movements in expectations which provide incentives in the present.

To disentangle both effects, we consider an average recursive plan constructed as follows. After solving for the recursive plan, we take the expected path of announcements which takes into account the fact that reputation will drift down over time, as the planner and the private sector understand. We then project this path onto the (a_0, ω, χ) space of our announcements, and solve for the continuation equilibrium after the announcement of this projected plan.

Table 2 provides a summarized comparison of our plans. In this parametrization, the gains from pre-announcing seem to be of the same magnitude as the gains from flexibility.

The recursive plan has a higher asymptote than reputational and K-equilibrium plans. This

Table 2: Inflation plans

Model	Ramsey	K-equilibrium	Avg. recursive plan	Recursive plan
Initial inflation	1.40%	1.63%	1.58%	1.58%
Long-run inflation	0%	0.44%	0.67%	0.67%
Value of loss function	0.3879	0.7552	0.7585	0.7557

reveals a second form of time inconsistency in the Ramsey plan. At time 0, the planner wants to promise to deliver low inflation in the future because this decreases the level of expected inflation in earlier periods. When the announcement is made sequentially, this gain evaporates and the planner attains a higher long-run level for inflation.

7. Concluding Remarks

This paper addresses an old question: can reputation be a substitute for commitment? We find that a simple model of reputation combined with imperfect control on the part of the government creates incentives for staying close to announced targets. The optimal policy after a plan has been announced trades off the benefits of surprise inflation against the possibility that a deviation becomes known to the public. In this way, the government's reputation becomes an important state variable in the problem of optimal policy under discretion.

Various characteristics of announced plans come to bear when determining the value of reputation. We find that a pervasive feature of optimal plans is gradualism. In anticipation of the continuation equilibrium, the planner finds it desirable to set itself up in situations where keeping its reputation is both easy and valuable. These are situations where announced inflation for the current period is higher than in the future. The resulting gradualism is therefore an artifact of incentives and not a reflection of inflation inertia. Understanding how the presence of sources of true inertia might interact with our results is one of our goals going forward.

The gradualist property of optimal plans holds at positive levels of reputation and also in the limit as initial reputation vanishes to zero. We interpret this limit case as a sensible refinement of the game between a rational government and the private sector.

Finally, we provide a comparison to models that stand between our reputational equilibria and more standard sustainable plans, based on some contributions to the literature. We argue that our model of reputation effectively operates by modifying the incentive constraint in the recursive version of a planning problem. When reputation is low, large option values of sticking to the plan are created, which become even greater when the plan is backloaded.

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